EXERCISES FOR THE "REFLECT" SOFTWARE PROGRAM

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INTRODUCTION

The REFLECT program is designed to simulate the sum over paths theory of quantum mechanics described by Richard P. Feynman (with Ralph Leighton) in the *book QED: The Strange Theory of Light and Matter* (Princeton University Press, 1985). In particular, REFLECT mimics partial reflection of light at the surfaces of glass.

A. READING FOR "REFLECT" EXERCISES

Chapter 1 of QED. On page 4 of this workbook is an alternative "Table of Contents" of the *QED* book by subject matter, compiled by Dan Styer. This table of contents is much more informative than the one provided in the QED book itself. Optional: Call up the REFLECT software and play with it to try out some of the ideas presented in the chapter. You will use this software in the exercises below.

DISCUSSION QUESTIONS ON THE READING

Answer the following discussion questions as you read (NOT to be turned in with homework -- but for class discussion).

A1. By what practical method(s) can one detect a single photon?

A2. How in the world can this SINGLE photon be BOTH reflected AND transmitted at the surface of the glass? or do both happen?

A3. A very thin sheet of glass yields zero reflection. How can this be, when the SAME FRACTION is reflected from both front and back surface?

A4. When the exploring photon is NOT reflected from a very thin sheet of glass, where does the photon go? Where can it be detected?

A5. Addition of arrows:
Arrow V is 2 units long and points north.
Arrow W is 1 unit long and points east.
Arrow X is 2.5 units long and points south.
Arrow Y is 3 units long and points west.
Arrow Z is 0.5 units long and points north.
When these arrows are added together, what is the DIRECTION and LENGTH of the resulting arrow? Is the resulting arrow different if the arrows are added together in a different order?

B. SECOND DETECTOR INSIDE GLASS

READING TO REVIEW FOR SECTION B: *QED,* Chapter 1 through page 19.

Call up the REFLECT program. Click on the button labeled SECOND DETECTOR and click somewhere INSIDE the glass in the leftmost panel. (ALL the questions in this section refer to the situation in which Detector B is INSIDE the glass -- same as Figure 2, page 17 of *QED*.) Now click on the START button.

What is going on? As the photon explores each path, the arrow rotates on the corresponding clock. The goal is to find the length of the arrow at each detector. The PROBABILITY that the photon will be detected at that detector is proportional to the square of the length of the arrow at that detector. (Feynman explains this on pages 24 and 25.) All partial reflection experiments—indeed, all the experiments of quantum mechanics!—can be described using these little rotating arrows. Is this weird or what?

Preliminary Calculations
In the display, the arrow on the "glass clock" is 0.98 as long as the original arrow on the original air clock. And the arrow for the reflected path is 0.20 as long as the original arrow on the air clock. Answer each of the following questions with a SINGLE number (a number may have more than one digit!) or a SINGLE word.

B1. What is the EXACT value of the square of 0.98? This is supposed to be the probability that the photon will be detected by Detector B under these circumstances.

B2. What is the EXACT value of the square of 0.2? This is supposed to be the probability that the photon will be detected by Detector A under these circumstances.

B3. What is the EXACT value of the sum of these squares? This is supposed to be the total probability that the photon will be detected EITHER by Detector A OR by Detector B under these circumstances.

B4. Do these probabilities add up to less than or more than unity?

B5. FIND a length for the so-called 0.98 arrow—accurate to four decimal places—such that the sum of probabilities DOES add up as nearly as possible to unity?

Computer Exercises
B6. Approximately how many revolutions does the original air clock make for the downward first portion of the path between Detector A and the top of the glass (for the original color red)? Answer should be an integer plus fraction, such as 4 1/8 revolutions.

B7. Place Detector B at DIFFERENT vertical positions, ALL of them inside the glass. Does the probability for *detection at A* change with the distance of B from the top surface of the glass?

B8. Continue the experiment placing Detector B at different vertical positions, ALL of them inside the glass. Does the probability for *detection at B* change with the distance of B from the top surface of the glass?

B9. Describe everything that changes concerning detection at B for different vertical locations of B inside the glass. (TEN words max. allowed to answer this question!)

B10. Does ANYTHING change concerning detection at A for different vertical locations of B inside the glass?

C. SINGLE DETECTOR A

READING FOR SECTION C: R. P. Feynman, *QED*, finish Chapter 1 (pages 20 thru 35).

ALL the questions in this section refer to the situation in which there is Detector A only (similar to Figures 10 and 11, pages 28 and 29 of *QED*).

Preliminary Calculations

In questions C1 thru C7, assume that in every transmission (air into glass or glass into air) the arrow length is multiplied by the factor 0.980 and in every reflection the arrow length is multiplied by the factor 0.200. Give answers as decimals to three decimal places.

C1. What is the length of the arrow for the exploring photon descending thru the glass before it reflects from the bottom surface of the glass?

C2. What is the length of the arrow passing upward through the glass after the first reflection from the LOWER surface of the glass?

C3. What is the length of the arrow after downward and upward passage through the glass and then transmission back into the air at its upper surface?

C4. What is the MAXIMUM possible length of the RESULTING ARROW when the arrow for the first path (simple reflection off the upper surface of the glass) is added to the arrow for the Second Path (answer to question C3)?

C5. Therefore what is the MAXIMUM PROBABILITY that the photon will be detected by Detector A?

C6. What is the MINIMUM length of the RESULTING ARROW when the arrow for the first path (simple reflection off the upper surface of the glass) is added to the arrow for the Second Path (answer to question C3)?

C7. Therefore, what is the MINIMUM PROBABILITY that the photon will be detected by Detector A? (to two significant figures)

Computer Exercises
Click on the button labeled DETECTOR A ONLY. Now click on the START button. All the following questions assume the default RED color of the photon. Use the GRAPH button to answer the following questions.

C8. What is the thickness of the glass closest to the value 100 pixels that gives a reflection maximum? Write down your answer.

C9. What is the thickness of the glass (larger or smaller, you choose) closest to your answer to question C8 that gives the next maximum? Write down your answer.

C10. What is the SMALLEST glass thickness (in pixels) for a MAXIMUM probability in Detector A?

C11. What is the SMALLEST glass thickness (in pixels) for a MINIMUM probability in Detector A?

C12. Are the glass thicknesses for maxima equally spaced, in pixels (plus or minus a pixel or two)? (DO NOT spend the time needed to trace out the entire curve. Work smart, not hard!)

D. DETECTOR B BELOW THE GLASS

READING FOR SECTION D: Feynman, *QED*, Chapter 2, pages 67 to Chapter 3, page 83.
Note: Before doing this reading or the following exercises, students have read Feynman *QED*, Chapter 2, pages 36 through 67 and carried out the Photon exercises for the OneParticle program.

ALL the questions in this section refer to the situation in which Detector B is below the glass (similar to Figure 4, page 20 and Figure 44, page 70 of *QED*).

Computer Exercises
D1. Does the photon move at the same speed in glass as in air? (Do not use the FAST setting to answer this, because in the FAST setting the computer runs as fast as it can, not at the scaled speed at which the photon should move.)

D2. What is the glass thickness closest to the value 100 pixels for which the detection probability at Detector B is a minimum? Write down your answer.

D3. On average, how far apart in glass thickness (pixels) are the minima of detection probability for Detector B? (You may want to answer question D4 at the same time as you answer this one. DO NOT take the time needed to trace out the entire curve. Work smart, not hard!)

D4. Are the probability minima for Detector B all the same depth (value below unity) for the range of glass thicknesses between 0 and 150 pixels? (DO NOT waste time . . .)

Computer Exercises
Questions D5 through D8 concern the situation for THE SAME glass thickness and DIFFERENT distances of Detector B below the glass—all for the color RED. Answer all questions yes or no.

D5. Does the *length* of the resulting arrow at Detector B differ for different distances of Detector B below the glass?

D6. Does the *rotation angle* of the resulting arrow differ for these different distances of Detector B below the glass?

D7. Does the *probability* that the photon will be detected in Detector B differ for these different distances of Detector B below the glass?

D8. If the probability at B is at a minimum for this glass thickness, will it be a minimum no matter where Detector B is placed below the glass?

E. CHANGES IN REFLECTION COEFFICIENT

Thus far we have assumed that the reflection coefficient has the value $R = 0.2$. Meaning: The magnitude of the rotating arrow after reflection from the upper or lower surface of the glass is 0.2 times the magnitude of the rotation arrow before reflection. This is the value that Feynman uses in his *QED* book. However, the value of the reflection coefficient is not a fundamental constant, but depends on the kind of glass.

You can change the value of the reflection coefficient R using the REFLECTION menu at the top of the screen. You can choose values between $R = 0.1$ and $R = 0.5$. (The default value is $R = 0.2$.) Then the machine computes the value of the transmission coefficient:

$$T = \left(1 - R^2\right)^{1/2}$$

this insures that the total probability of reflection plus the probability of transmission add up to unity:

$$R^2 + T^2 = 1$$

For the following questions, assume that the Detector B is below the glass.

E1. *Predict:* Suppose you find a thickness of the glass that gives maximum transmission for reflection coefficient R = 0.2. Will the same thickness of glass give maximum transmission when you change the reflection coefficient to the value R = 0.5?

E2. *Verify:* Use the REFLECT program to determine whether your prediction in E1 is correct or incorrect.

Some students have discovered that when the reflection coefficient has the value R = 0.5 the sum of the transmission probability plus the reflection probability does not add up to unity when they use the REFLECT program. They have an explanation for this result: We have arbitrarily limited the number of internal reflections, so that:

Final arrow at Detector A is the result of adding two little arrows: (1) the little arrow due to a single reflection at the upper surface of the glass, plus (2) the little arrow due to one round trip in the glass (one internal reflection at the lower surface).

Final arrow at Detector B is also the result of adding two little arrows: (1) the little arrow due to direct transmission through the glass, plus (2) the little arrow due to two internal reflections in the glass (one internal reflection at the lower surface and one at the upper surface).

The students say that, with the default reflection coefficient R = 0.2, these alternative paths are enough: The little arrows after further internal reflections would be so small as to add negligible amounts to the final arrows at Detector A or Detector B.

BUT, these students continue, with a larger reflection coefficient, the little arrows after further internal reflections remain large enough to affect significantly the final arrows at Detector A and Detector B.

E3. In the REFLECT program, set the reflection coefficient to the value R = 0.5. Find a glass thickness for which transmission is a *maximum*. Write down the resulting probability shown for transmission and reflection. Do these two probabilities add up to more than unity or less than unity?

E4. With the same value of the reflection coefficient, find a glass thickness for which transmission is a *minimum*. Write down the resulting probability shown for transmission and reflection. Do these two probabilities add up to more than unity or less than unity?

E5. One of our investigating students objects: “The total of reflection probability plus transmission probability in the program can never be greater than unity, because further reflections will always ADD to the resulting arrows computed by the existing program.” Is this student correct or incorrect? Explain your reasoning.