CHAPTER 3

SAME LAWS FOR ALL

The name relativity theory was an unfortunate choice: The relativity of space and time is not the essential thing, which is the independence of laws of Nature from the viewpoint of the observer.

Arnold Sommerfeld

3.1 THE PRINCIPLE OF RELATIVITY

fundamental science needs only a closed room

How do you know you are moving? Or at rest? In a car, you pause at a stoplight. You see the car next to you easing forward. With a shock you suddenly realize that, instead, your own car is rolling backward. On an international flight you watch a movie with the cabin shades drawn. Can you tell if the plane is traveling at minimum speed or full speed? In an elaborate joke, could the plane actually be sitting still on the runway, engines running? How would you know?

Principle of Relativity: With shades drawn you cannot tell your speed

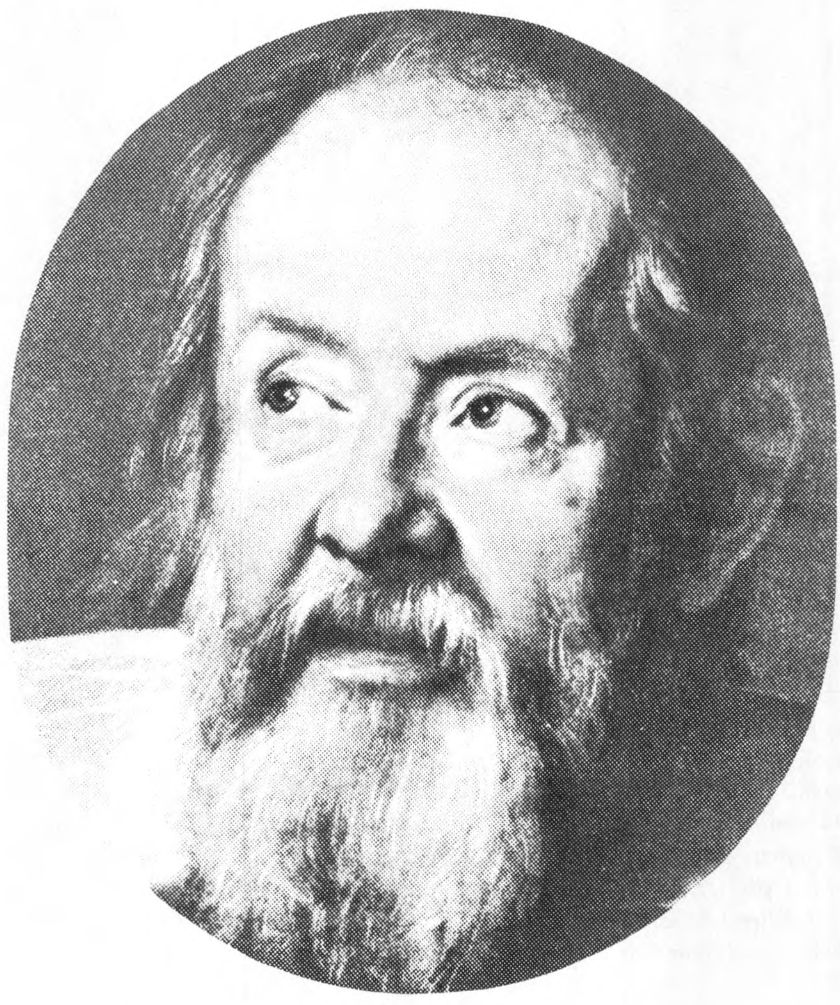
Everyday observations such as these form the basis for a conjecture that Einstein raised to the status of a postulate and set at the center of the theory of special relativity. He called it the **Principle of Relativity.** Roughly speaking, the Principle of Relativity says that without looking out the window you cannot tell which reference frame you are in or how fast you are moving.

Galileo Galilei made the first known formulation of the Principle of Relativity. Listen to the characters in his book:

Galileo: First known formulation of Principle of Relativity

> SALVATIUS: Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any

of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air . . .



GALILEO GALILEI

Pisa, February 15, 1564—Arcetri, near Florence, January 8, 1642

"My portrait is now finished, a very good likeness, by an excellent hand."
—*September 22, 1635*

* * *

"If ever any persons might challenge to be signally distinguished for their intellect from other men, Ptolemy and Copernicus were they that had the honor to see farthest into and discourse most profoundly of the World's systems."

* * *

"My dear Kepler, what shall we make of all this? Shall we laugh, or shall we cry?"

* * *

"When shall I cease from wondering?"

SAGREDUS: Although it did not occur to me to put these observations to the test when I was voyaging, I am sure that they would take place in the way you describe. In confirmation of this I remember having often found myself in my cabin wondering whether the ship was moving or standing still; and sometimes at a whim I have supposed it to be going one way when its motion was the opposite . . .

The Galilean Principle of Relativity is simple in this early formulation, yet not as simple as it might be. In what way is it simple? Physics looks the same in a ship moving uniformly as in a ship at rest. Relative uniform motion of the two ships does not affect the laws of motion in either ship. A ball falling straight down onto one ship appears from the other ship to follow a parabolic course; a ball falling straight down onto that second ship also appears to follow a parabolic course when observed from the first ship. The simplicity of the Galilean Principle of Relativity lies in the equivalence of the two Earthbound frames and the symmetry between them.

Extension of Galileo's reasoning from ship to spaceship

In what way is this simplicity not as great as it might be? In Galileo's account the frames of reference are not yet free-float (inertial). To make them so requires only a small conceptual step: from two uniformly moving sea-going ships to two unpowered spaceships. Then up and down, north and south, east and west, all become alike. A ball untouched by force undergoes no acceleration. Its motion with respect to one spaceship is as uniform as it is with respect to the other. This identity of the law of free motion in all inertial reference frames is what one means today by the Galilean Principle of Relativity.

Galileo could not by any stretch of the imagination have asked his hearer to place himself in a spaceship in the year 1632. Yet he could have described the greater simplicity of physics when viewed from such a vantage point. Bottles, drops of water, and all the other test objects float at rest or move at uniform velocity. The zero acceleration of every nearby object relative to the spaceship would have been intelligible to Galileo of all people. Who had established more clearly than he that relative to Earth all nearby objects have a common acceleration?

Einstein's Principle of Relativity is a generalization of such experiments and many other kinds of experiments, involving not only mechanics but also electromagnetism, nuclear physics, and so on.

Principle of Relativity

All the laws of physics are the same in every free-float (inertial) reference frame.

Einstein's Principle of Relativity says that once the laws of physics have been established in one free-float frame, they can be applied without modification in any other free-float frame. Both the mathematical form of the laws of physics and the numerical values of basic physical constants that these laws contain are the same in every free-float frame. So far as concerns the laws of physics, all free-float frames are equivalent.

We can tell where we are on Earth by looking out of the window. Where we are in the Milky Way we can tell by the configuration of the Big Dipper and other constellations. How fast and in what direction we are going through the larger framework of the universe we measure with a set of microwave horns pointed to pick up the microwave radiation streaming through space from all sides. But now exclude all information from outside. Screen out all radiation from the heavens. Pull down the window shade. Then do whatever experiment we will on the movement and collision of particles and the action of electric and magnetic forces in whatever free-float frame we please. We find not the slightest difference in the fit to the laws of physics between measurements made in one free-float frame and those made in another. We arrive at the Principle of Relativity in its negative form:

Principle of Relativity, negative form

No test of the laws of physics provides any way whatsoever to distinguish one free-float frame from another.

BOX 3-1

THE PRINCIPLE OF RELATIVITY RESTS ON EMPTINESS!

In his paper on special relativity, Einstein says, "We will raise this conjecture (whose intent will from now on be referred to as the 'Principle of Relativity') to a postulate . . . " Is the Principle of Relativity just a postulate? All of special relativity rests on it. How do we know it is true? What lies behind the Principle of Relativity?

This is a philosophical question, not a scientific one. You will have your own opinion; here is ours. We think the Principle of Relativity as used in special relativity rests on one word: emptiness.

Space is empty; there are no kilometer posts or mileposts in space. Do you want to measure distance and time? Then set up a latticework of meter sticks and clocks. Pace off the meter sticks, synchronize the clocks. Use the latticework to carry out your measurements. Discover the laws of physics. This latticework is your construction, not Nature's. Do not ask Nature to choose your latticework in preference to the similar latticework that I have constructed. Why not? Because space is empty. Space accommodates both of us as we go about our constructions and our investigations. But it does not choose either one of us in preference to the other. How can it? Space is empty. Nothing whatever can distinguish your latticework from mine. If we decide in secret to exchange latticeworks, Nature will never be the wiser! It follows that whatever laws of physics you discover employing your latticework must be the same laws of physics I discover using my latticework. The same is true even when our lattices move relative to one another. Which one of us is at rest? There is no way to tell in empty space! This is the Principle of Relativity.

But is space *really* empty? "Definitely not!" says modern quantum physics. "Space is a boiling cauldron of virtual particles. To observe this cauldron,

3.2 WHAT IS *NOT* THE SAME IN DIFFERENT FRAMES

not the same: space separations, time separations, velocities, accelerations, forces, fields

Space and time separations not the same in different frames

Notice what the Principle of Relativity does *not* say. It does not say that the time between two events is the same when measured from two different free-float frames. Neither does it say that space separation between the two events is the same in the two frames. Ordinarily neither time nor space separations are the same in the two frames.

The catalog of differences between readings in the two frames does not end with laboratory and rocket records of pairs of events. Physics to the Greeks meant the science of change and so it does to us today. Motion gives us a stream of events, for example the blinks of a firefly or the pulses of a sparkplug flashing as it moves. These flashes trace out the sparkplug's trajectory. Record the positions of two sequential sample regions of space much smaller than the proton. Carry out this sampling during times much shorter than the time it takes light to cross the diameter of the proton." These words are familiar or utterly incomprehensible, depending on the amount of our experience with physics. In either case, we can avoid dealing with the "boiling cauldron of virtual particles" by observing events that are far apart compared with the dimensions of the proton, events separated from one another by times long compared with the time it takes light to cross the diameter of the proton.

In the realm of classical (nonquantum) physics is space really empty? "Of course not!" says modern cosmology. "Space is full of stars and dust and radiation and neutrinos and white dwarfs and neutron stars and (many believe) black holes. To observe these structures, sample regions of space comparable in size to that of our galaxy. These structures evolve and move with respect to one another in times comparable to millions of years."

So we choose regions far from massive structures, avoid dust, ignore neutrinos and radiation, and measure events that take place close together in time compared with a million years.

Notice that for the very small and also for the very large, the "regions" described span both space and time — they are regions of *spacetime*. "Emptiness" refers to spacetime. Therefore we should have said from the beginning, "*Spacetime* is empty" — except for us and our apparatus — with limitations described above.

In brief, we can find "effectively empty" regions of spacetime of spatial extent quite a few orders of magnitude larger and smaller than dimensions of our bodies and of time spread quite a few orders of magnitude longer and shorter than times that describe our reflexes. In spacetime regions of this general size, empty spacetime can be found. In empty spacetime the Principle of Relativity applies. Where the Principle of Relativity applies, special relativity correctly describes Nature.

spark emissions in the laboratory frame. Record also the laboratory time between these sparks. Divide the change in position by the increase in time, yielding the laboratory-measured velocity of the sparkplug.

Spark events have identities that rise above all differences between reference frames. These events are recorded not only in the laboratory but also by recording devices and clocks in the rocket latticework. From the printouts of the recorders in the rocket frame we read off rocket space and time separations between sequential sparks. We divide. The quotient gives the rocket-measured velocity of the sparkplug. But both the space separation and the time separation between events, respectively, are ordinarily different for the rocket frame than for the laboratory frame. Therefore the rocket-measured velocity of the sparkplug is different from the laboratory-measured velocity of that sparkplug. Same world. Same motion. Different records of that motion. Figures for velocity that differ between rocket and laboratory.

Velocity not the same

Acceleration not the same

Apply force to a moving object: Its velocity changes; it accelerates. Acceleration is the signal that force is being applied. Two events are enough to reveal velocity; three reveal change in velocity, therefore acceleration, therefore force. The laboratory observer reckons velocity between the first and second events, then he reckons velocity

BOX 3-2

THE SPEED OF LIGHT

A "fundamental constant of nature"?
Or a mere factor of conversion between two units of measurement?

METERS AND MILES IN THE PARABLE OF THE SURVEYORS

Meter?
Originally (adopted France, 1799) one ten-millionth of the distance along the surface of Earth from its equator to its pole (in a curved line of latitude passing through the center of Paris).

Mile?
Originally one thousand *paces*—double step: right to left to right—of the Roman soldier.

Modern conversion factor?
1609.344 meters per mile.

Authority for this number?
Measures of equator-to-pole distance eventually (1799 to today) lagged in accuracy compared to laboratory measurement of distance. So the platinum meter rod at Sèvres, Paris, approximating one ten-millionth of that distance, for awhile became—in and by itself—the standard of distance. During that time the British Parliament and the United States Congress redefined the inch to be *exactly* 2.54 centimeters. This decree made the conversion factor (5280 feet/mile) times (12 inches/foot) times (2.54 centimeters/inch) times (1/100 of a meter per centimeter) equal to 1609.344 meters per mile—exactly!

A fundamental constant of nature?
Hardly! Rather, the work of two centuries of committees.

SECONDS AND METERS IN SPACETIME

Second?
Originally 1/24 of 1/60 of 1/60 of the time from high noon one day to high noon the next day. Since 1967, "The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the fundamental state of the atom cesium 133."

Meter?
Definition evolved from geographic to platinum meter rod to today's "One meter is the distance traveled by light, in vacuum, in the fraction 1/299,792,458 of a second."

Modern conversion factor?
299,792,458 meters per second.

Authority for this number?
Meeting of General Conference on Weights and Measures, 1983. In the accepted definition of the meter important changes took place over the years, and likewise in the definition of the second. With the 1983 definition of the meter these two streams of development have merged. What used to be understood as a measurement of the speed of light is understood today as two ways to measure separation in spacetime.

A fundamental constant of nature?
Hardly! Rather, the work of two centuries of committees.

---

between the second and third events. Subtracting, he obtains the change in velocity. From this change he figures the force applied to the object.

The rocket observer also measures the motion: velocity between the first and second events, velocity between second and third events; from these the change in velocity; from this the force acting on the object. But the rocket-observed velocities are not equal to the corresponding laboratory-observed velocities. The *change* in velocity also differs in the two frames; therefore the computed *force* on the object is different for

Force not the same

Commentary

Is the distance from Earth's equator to its pole a fundamental constant of nature? No. Earth is plastic and ever changing. Is the distance between the two scratches on the standard meter bar constant? No. Oxidation from decade to decade slowly changes it. Experts in the art and science of measurement move to ever-better techniques. They search out an ever-better object to serve as benchmark. Via experiment after experiment they move from old standards of measurement to new. The goals? Accuracy. Availability. Dependability. Reproducibility.

Make a better measurement of the speed of light. Gain in that way better knowledge about light? No. Win instead an improved value of the ratio between one measure of spacetime interval, the meter, and another such measure, the second—both of accidental and historical origin? Before 1983, yes. Since 1983, no. Today the meter is *defined* as the distance light travels in a vacuum in the fraction 1/299,792,458 of a cesium-defined second. The two great streams of theory, definition, and experiment concerning the meter and the second have finally been unified.

What will be the consequence of a future, still better, measuring technique? Possibly it will shift us from the cesium-atom-based second to a pulsar-based second or to a still more useful standard for the second. But will that improvement in precision change the speed of light? No. Every past International Committee on Weights and Measures has operated on the principle of minimum dislocation of standards; we have to expect that the speed of light will remain at the decreed figure of 299,792,458 meters per second, just as the number of meters in the mile will remain at 1609.344. Through the fixity of this conversion factor c, any substantial improvement in the accuracy of defining the second will bring with it an identical improvement in the accuracy of defining the meter.

Is 299,792,458 a fundamental constant of nature? Might as well ask if 5280 is a fundamental constant of nature!

rocket observer and laboratory observer. The Principle of Relativity does not deny that the force acting on an object is different as reckoned in two frames in relative motion.

An electric field or a magnetic field or some combination of the two, acting on the electron, is the secret of action of many a device doing its quiet duty day after day in home, factory, or car. An electromagnetic force acting on an electron changes its velocity as it moves from event P to event Q and from Q to R. Laboratory and rocket observers do not agree on this change in velocity. Therefore they do not agree on the

Electric and magnetic fields not the same value of the force that changes that velocity. Nor, finally, do they agree on the magnitudes of the electric and magnetic fields from which the force derives.

In brief, figures for electric and magnetic field strengths, for forces, and for accelerations agree no better between rocket and laboratory observers than do figures for velocity. The Principle of Relativity does not deny these differences. It celebrates them. It explains them. It systematizes them.

3.3 WHAT *IS* THE SAME IN DIFFERENT FRAMES

the same: physical laws, physical constants in those laws

Laws of physics the same in different frames

Different values of some physical quantities between the two frames? Yes, but identical physical *laws!* For example, the relation between the force acting on a particle and the change in velocity per unit time of that particle follows the same law in the laboratory frame as in the rocket frame. The force is not the same in the two frames. Neither is the change in velocity per unit time the same. But the law that relates force and change of velocity per unit time is the same in each of the two frames. All the laws of motion are the same in the one free-float frame as in the other.

Not only the laws of motion but also the laws of electromagnetism and all other laws of physics hold as true in one free-float frame as in any other such frame. This is what it means to say, "No test of the laws of physics provides any way whatsoever to distinguish one free-float frame from another."

Fundamental constants the same

Deep in the laws of physics are numerical values of fundamental physical constants, such as the elementary charge on the electron and the speed of light. The values of these constants must be the same as measured in overlapping free-float frames in relative motion; otherwise these frames could be distinguished from one another and the Principle of Relativity violated.

Speed of light the same

One basic physical constant appears in the laws of electromagnetism: the speed of light in a vacuum, $c = 299{,}792{,}458$ meters per second. According to the Principle of Relativity, this value must be the same in all free-float frames in uniform relative motion. Has observation checked this conclusion? Yes, many experiments demonstrate it daily and hourly in every particle-accelerating facility on Earth. Nevertheless, it has taken a long time for people to become accustomed to the apparently absurd idea that there can be one special speed, the speed of light, that has the same value measured in each of two overlapping free-float frames in relative motion.

Values of the speed of light as measured by laboratory and by rocket observer turn out identical. This agreement has cast a new light on light. Its speed rates no longer as a constant of nature. Instead, today the speed of light ranks as mere conversion factor between the meter and the second, like the factor of conversion from the centimeter to the meter. The value of this conversion factor has now been set by decree and the meter defined in terms of it (Box 3.2). This decree *assumes* the invariance of the speed of light. No experimental result contradicts this assumption.

In 1905 the Principle of Relativity was a shocking heresy. It offended most people's intuition and common-sense way of looking at Nature. Consequences of the Principle of Relativity are tried out every day in many experiments where it is continually under severe test. Never has this Principle been verified to lead to a single incorrect experimental prediction.

SAMPLE PROBLEM 3-1

EXAMPLES OF THE PRINCIPLE OF RELATIVITY

Two overlapping free-float frames are in uniform relative motion. According to the Principle of Relativity, which of the quantities on the following list must *necessarily* be the same as measured in the two frames? Which quantities are *not* necessarily the same as measured in the two frames?

a. numerical value of the speed of light in a vacuum

b. speed of an electron

c. value of the charge on the electron

d. kinetic energy of a proton (the nucleus of a hydrogen atom)

e. value of the electric field at a given point

f. time between two events

g. order of elements in the periodic table

h. Newton's First Law of Motion ("A particle initially at rest remains at rest, and . . . ")

SOLUTION

a. The speed of light IS necessarily the same in the two frames. This is one of the central tenets of the Principle of Relativity and a basis of the theory of relativity.

b. The speed of an electron IS NOT necessarily the same in the two frames. Determining the speed of a particle depends on space and time measurements between events — such as flashes emitted by the particle. Space and time separations between events, respectively, can be measured to be different for observers in relative motion. So the speed — ratio of distance covered to time elapsed — can be different.

c. The value of the charge on the electron IS necessarily the same in the two frames. Suppose that the charge had one value for the laboratory frame and progressively smaller values for rocket frames moving faster and faster relative to the laboratory frame. Then we could detect the "absolute velocity" of the frame we are in by measuring the charge on the electron. But this violates the Principle of Relativity. Therefore the charge on the electron must have the same value in all free-float frames.

d. The kinetic energy of a proton IS NOT necessarily the same in the two frames. The value of its kinetic energy depends on the speed of the proton. But speed is not necessarily the same as measured in the two frames (**b**).

e. The value of the electric field at a given point IS NOT necessarily the same in the two frames. The argument is indirect but inescapable: The electric field is measured by determining the force on a test charge. Force can be measured by change in velocity that the force imparts to a particle of known mass. But the velocity — and the change in velocity — of a particle can be *different* for observers in relative motion (**b**). Therefore the electric field may be different for observers in relative motion.

f. The time between two events IS NOT necessarily the same in the two frames. This is a direct result of the invariance of the interval (Chapter 1 and Section 3.7).

SAMPLE PROBLEM 3-1

g. The order of elements in the periodic table by atomic number IS necessarily the same in the two frames. For suppose that the atomic number (the number of protons in the nucleus) were smaller for helium than for uranium in the laboratory frame but greater for helium than for uranium in the rocket frame. Then we could tell which frame we were in by comparing the atomic numbers of helium and uranium.

h. Newton's First Law of Motion IS necessarily the same in the two frames. Newton's First Law is really a definition of the inertial (free-float) frame. We assume that all laboratory and rocket frames are inertial.

3.4 RELATIVITY OF SIMULTANEITY

"same time"? ordinarily true for only one frame!

The Principle of Relativity directly predicts effects that initially seem strange—even weird. Strange or not, weird or not; logical argument demonstrates them and experiment verifies them. One effect has to do with simultaneity: Let two events occur separated in space along the direction of relative motion between laboratory and rocket frames. These two events, even if simultaneous as measured by one observer, cannot be simultaneous as measured by both observers.

Einstein demonstrated the relativity of simultaneity with his famous Train Paradox. (When Einstein developed the theory of special relativity, the train was the fastest common carrier.) Lightning strikes the front and back ends of a rapidly moving train, leaving char marks on the train and on the track and emitting flashes of light that travel forward and backward along the train (Figure 3-1). An observer standing on the ground halfway between the two char marks on the track receives the two light flashes at the same time. He therefore concludes that the two lightning bolts struck the track at the same time—with respect to him they fell simultaneously.

Train Paradox: Two lightning bolts strike simultaneously for ground observer

A second observer rides in the middle of the train. From the viewpoint of the observer on the ground, the train observer moves toward the flash coming from the front of the train and moves away from the flash coming from the rear. Therefore the train observer receives the flash from the front of the train first.

This is just what the train observer finds: The flash from the front of the train arrives at her position first, the flash from the rear of the train arrives later. But she can verify that she stands equidistant from the front and rear of the train, where she sees char marks left by the lightning. Moreover, using the Principle of Relativity, she knows that the speed of light has the same value in her train frame as for the ground observer (Section 3.3 and Box 3-2), and is the same for light traveling in both directions in her frame. Therefore the arrival of the flash first from the front of the train leads her to conclude that the lightning fell first on the front end of the train. For her the lightning bolts did not fall simultaneously. (To allow the train observer to make only measurements with respect to the train, forcing her to ignore Earth, let the train be a cylinder without windows—in other words a spaceship!)

Two lightning bolts *do not* strike simultaneously for train observer

Did the two lightning bolts strike the front and the back of the train simultaneously? Or did they strike at different times? Decide!

Strange as it seems, there is no unique answer to this question. For the situation described above, the two events are simultaneous as measured in the Earth frame; they

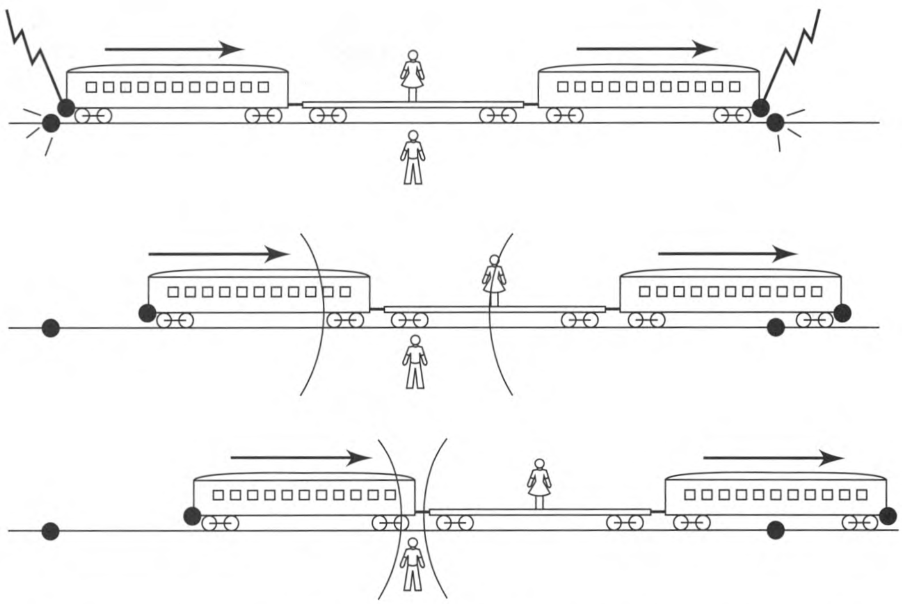


FIGURE 3-1. ***Einstein's Train Paradox illustrating the relativity of simultaneity. Top:*** *Lightning strikes the front and back ends of a moving train, leaving char marks on both track and train. Each emitted flash spreads out in all directions.* ***Center:*** *Observer riding in the middle of the train concludes that the two strokes are not simultaneous. Her argument: "(1) I am equidistant from the front and back char marks on the train. (2) Light has the standard speed in my frame, and equal speed in both directions. (3) The flash arrived from the front of the train first. Therefore, (4) the flash must have left the front of the train first; the front lightning bolt fell before the rear lightning bolt fell. I conclude that the lightning strokes* were not simultaneous." ***Bottom:*** *Observer standing by the tracks halfway between the char marks on the tracks concludes that the strokes* were simultaneous, *since the flashes from the strokes reach him at the same time.*

Simultaneity is relative

are not simultaneous as measured in the train frame. We say that the simultaneity of events is, in general, *relative,* different for different frames. Only in the special case of two or more events that occur at the same point (or in a plane perpendicular to the line of relative motion at that point — see Section 3.6) does simultaneity in the laboratory frame mean simultaneity in the rocket frame. When the events occur at different locations along the direction of relative motion, they cannot be simultaneous in both frames. This conclusion is called the **relativity of simultaneity.**

The relativity of simultaneity is a difficult concept to understand. Almost without exception, every puzzle and apparent paradox used to "disprove" the theory of relativity hinges on some misconception about the relativity of simultaneity.

3.5 LORENTZ CONTRACTION OF LENGTH

space separation between two length-measuring events? disagreement!

Length of a rod = separation between simultaneous sparks at its two ends

How do we measure the length of a moving rod — the distance between one end and the other end? One way is to use our latticework of clocks to mark the location of the two ends at the same time. But when the rod lies along the direction of relative motion, someone riding with the rod does not agree that our marking of the positions of the two ends occurs at the same time (Section 3.4). The relativity of simultaneity tells us that rocket and laboratory observers disagree about the simultaneity of two events (firecrackers exploding at the two ends of the rod) that occur at different locations along the direction of relative motion. Therefore the two observers disagree about whether or not a valid measurement of length has taken place.

Disagree about simultaneity? Then disagree about length.

Go back to the Train Paradox. For the observer standing on the ground, the two lightning bolts strike the front and back of the train at the same time. Therefore for him the distance between the char marks on the track constitutes a valid measure of the length of the train. In contrast, the observer riding on the train measures the front lightning bolt to strike first, the rear bolt later. The rider on the train exclaims to her Earth-based colleague, "See here! Your front mark was made before the back mark — since the flash from the front reached me (at the middle of the train) before the flash from the back reached me. Of course the train moved during the time lapse between these two lightning strikes. By the time the stroke fell at the back of the train, the front of the train had moved well past the front char mark on the track. Therefore your measurement of the length of the train is too small. The train is really longer than you measured."

There are other ways to measure the length of a moving rod. Many of these methods lead to the same result: the space separation between the ends of the rod is less as measured in a frame in which the rod is moving than as measured in a frame in which the rod is at rest. This effect is called **Lorentz contraction.** Section 5.8 examines the Lorentz contraction quantitatively.

Suppose we agree to measure the length of a rod by determining the position of its two ends at the same time. Then an observer for whom the rod is at rest measures the rod to be longer than does any other observer. This "rest length" of the rod is often called its **proper length.**

You keep using the word "measure." Occasionally you say "observe." You never talk about that most delicate, sensitive, and refined of our five senses: sight. Why not just look *and* see *these remarkable relativistic effects?*

We have been careful to say that the relativity of simultaneity and the Lorentz contraction are *measured,* not *seen* with the eye. *Measurement* employs the latticework of rods and clocks that constitutes a free-float frame. As mentioned in Chapter 2, seeing with the eye leads to confused images due to the finite speed of light. Stand in an open field in the southern hemisphere as Sun sets in the west and full Moon rises in the east: You see Moon as it was 1.3 seconds ago, Sun as it was eight minutes ago, the star Alpha Centauri (nearest star visible to the naked eye) as it was 4.34 years ago, the Andromeda nebula as it was *2 million years ago*—you see them all *now.* Similarly, light from the two separated ends of a speeding rod typically takes different times to reach your eye. This relative time delay results in visual distortion that is avoided when the location of each end is recorded locally, with zero or minimal delay, by the nearest lattice clock. Visual appearance of rapidly moving objects is itself an interesting study, but for most scientific work it is an unnecessary distraction. To avoid this kind of confusion we set up the free-float latticework of synchronized recording clocks and insist on its use—at least in principle!

Aha! Then I have caught you in a contradiction. Figure 3-1 shows lightning flashes and trains. Is this not a picture of what we would see with our eyes?

No. Strictly speaking, each of the three "pictures" in Figure 3-1 summarizes where parts of the train are as recorded by the Earth latticework of clocks at a given instant of Earth time. The position of each light flash at this instant is also recorded by the clocks in the lattice. The summary of data is then given to a draftsman, who draws the picture for that Earth time. To distinguish such a drafted picture from the visual view, we will often refer to it as a **plot.** For example, Figure 3-1 (top) is the Earth plot at the time when lightning bolts strike the two ends of the train.

Actually, all three plots in Figure 3-1 show approximately what you see through a telescope when you are very far from the scene in a direction perpendicular to the direction of motion of the train and at a position centered on the action. At such a remote location, light from all parts of the scene takes approximately equal times to reach your eye, so you would see events and objects at approximately the same time according to Earth clocks. Of course, you receive this information later than it actually occurs because of the time it takes light to reach you.

3.6 INVARIANCE OF TRANSVERSE DIMENSION

"faster" does not mean "thinner" or "fatter"

A rocket ship makes many trips past the laboratory observer, each at successively higher speed. For each new and greater speed of the rocket, the laboratory observer measures its length to be shorter than it was on the trip before. This observed contraction is **longitudinal**—along its direction of motion. Does the laboratory observer also measure contraction in the **transverse** dimension, perpendicular to the direction of relative motion? In brief, is the rocket measured to get thinner as well as shorter as it moves faster and faster?

Transverse dimension same for laboratory and rocket observers

The answer is No. This is confirmed experimentally by observing the width of electron and proton beams traveling in high-energy accelerators. It is also easily demonstrated by simple thought experiments.

Speeding-Train Thought Experiment: Return to Einstein's high-speed railroad train seen end-on (Figure 3-2). Suppose the Earthbound observer measures the train to get thinner as it moves faster. Then for the Earth observer the right and left wheels of the train would come closer and closer together as the train speeds up, finally slipping off *between the tracks* to cause a terrible wreck. In contrast, the train observer regards herself as at rest and the tracks as speeding by in the opposite direction. If she

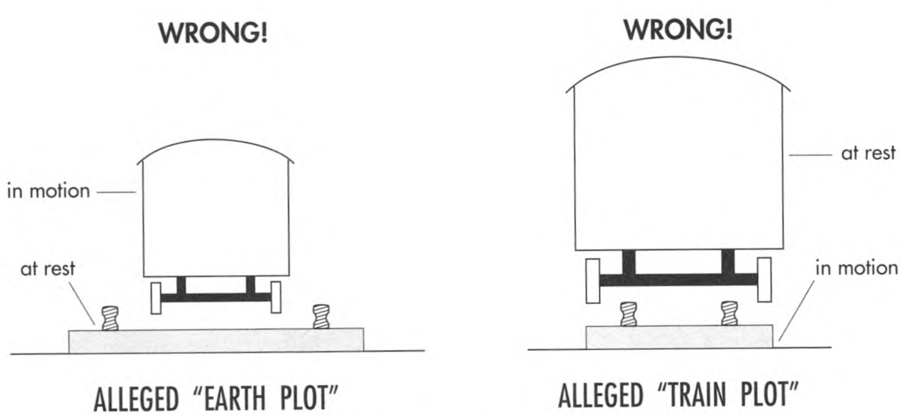


FIGURE 3-2. ***Two possible alternatives (both wrong!) if the moving train is measured to shrink transverse to its direction of motion.*** *The* ***"Earth plot"*** *assumes the speeding train to be measured as getting thinner with increasing speed. The train's wheels would slip off* between the tracks. *The* ***"train plot"*** *of the same circumstance assumes the speeding rails to be measured as getting closer together. In this case the wheels would slip off* outside the tracks. *But this is a contradiction. Therefore the wheel separation —and the transverse dimensions of train and track—must be* invariant, *the same for all free-float observers moving along the track. (If you think that the actual transverse contraction might be too small to cause a wreck for the train shown, assume that both the wheels and the track are knife edges; the same argument still applies.)*

measures the speeding tracks to get closer together as they move faster and faster, the train wheels will slip off *outside the tracks,* also resulting in a wreck. But this is absurd: the wheels cannot end up between the tracks and outside the tracks under the same circumstances. Conclusion: High speed leads to no measured change in transverse dimensions — no observed thinning or fattening of fast objects. We are left with the conclusion that high relative speed affects the measured values of longitudinal dimensions but not transverse dimension: a welcome simplification!

Thought experiments demonstrate invariance of transverse dimension

Speeding-Pipes Thought Experiment: Start with a long straight pipe. Paint one end with a checkerboard pattern and the other end with stripes. Cut out and discard the middle of the pipe, leaving only the painted ends. Now hurl the ends toward each other, with their cylindrical axes lying along a common line parallel to the direction of relative motion (Figure 3-3). Suppose that a moving object is measured to be thinner. Then someone riding on the checkerboard pipe will observe the striped pipe to pass inside her cylinder. *All* observers — everyone looking from the side — will see a checkerboard pattern. In contrast, someone riding on the striped pipe will observe the checkerboard pipe to pass inside his cylinder. In this case, all observers will see a striped pattern. Again, this is absurd: All observers must see stripes, or all must see checkerboard. The only tenable conclusion is that speed has no measurable effect on transverse dimensions and the pipe segments will collide squarely edge on.

A simple question leads to an even more fundamental argument against the difference of transverse dimensions of a speeding object as observed by different free-float observers in relative motion: *About what axis* does the contraction take place?

We try to define an "axis of shrinkage" parallel to the direction of relative motion. Can we claim that a speeding pipe gets thinner by shrinking uniformly toward an "axis of shrinkage" lying along its center? Then what happens when two pipe segments move along their lengths, side by side as a pair? Does each pipe shrink separately, causing the clear space between them to *increase?* Or does the combination of both pipes contract toward the line midway between them, causing the clear space between them to *decrease?* Is the answer different if one pipe is made of lead and the other one of paper? Or if one pipe is entirely in our imagination?

There is no logically consistent way to define an "axis of shrinkage." Given the direction of relative motion of two objects, we cannot select uniquely an "axis of shrinkage" from the infinite number of lines that lie parallel in this direction. For each different choice of axis a different pattern of distortions results. But this is logically intolerable. The only way out is to conclude that there is no transverse shrinkage at all (and, by a similar argument, no transverse expansion).

The above analysis leads to conclusions about events as well as about objects. A set of explosions occurs around the perimeter of the checkerboard pipe. More: These explosions occur simultaneously in this checkerboard frame. Then these events are simultaneous also in the striped frame. How do we know? By symmetry! For suppose the explosions were *not* simultaneous in the striped frame. Then which one of these

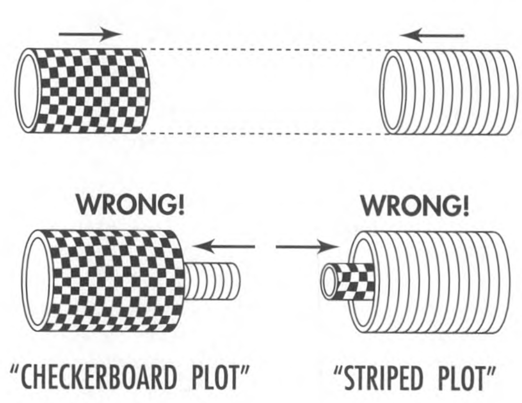


FIGURE 3-3. *Two identical-size pipe segments hurtle toward each other along a common centerline. What will happen when they meet? Here are two possible alternatives (both wrong!) if a moving object is observed to shrink transverse to direction of motion. Which pipe passes inside the other? The impossibility of a consistent answer to this question leads to the conclusion that neither pipe can be measured to change transverse dimension.*

events would occur first in the striped frame? The one on the right side of the pipe or the one on the left side of the pipe? But "left" and "right" cannot be distinguished by means of any physical effect: Each pipe is cylindrically symmetric. Moreover, space is the same in all directions — space is **isotropic,** the same to right as to left. So neither the event on the right side nor the event on the left side can be first. They must be simultaneous. The same argument can be made for events at the "top" and "bottom" of the pipe, and for every other pair of events on opposite sides of the pipe. Conclusion: If the explosions are simultaneous in the checkerboard frame, they must also be simultaneous in the striped frame.

"Same time" agreed on for events separated only transverse to relative motion

We make the following summary conclusions about dimensions transverse to the direction of relative motion:

> **Dimensions of moving objects transverse to the direction of relative motion are measured to be the same in laboratory and rocket frames (invariance of transverse distance).**
>
> **Two events with separation only transverse to the direction of relative motion and simultaneous in either laboratory or rocket frame are simultaneous in both.**

3.7 INVARIANCE OF THE INTERVAL PROVED

laboratory and rocket observers agree on something important

The Principle of Relativity has a major consequence. It demands that the spacetime interval have the same value as measured by observers in every overlapping free-float frame; in brief, it demands "invariance of the interval." Proof? Plan of attack: Determine the separation in space and the separation in time between two events, E and R, in the rocket frame. Then determine the quite different space and time separations between the same two events as measured in a free-float laboratory frame. Then look for — and find — what is invariant. It is the "interval." Now for the details (Figures 3-4 and 3-5).

Principle of Relativity leads to invariance of spacetime interval

Event E we take to be the reference event, the emission of a flash of light from the central laboratory and rocket reference clocks as they coincide at the zero of time (Section 2.6). The path of this flash is tracked by the recording clocks in the rocket lattice. Riding with the rocket, we examine that portion of the flash that flies straight "up" 3 meters to a mirror. There it reflects straight back down to the photodetector located at our rocket reference clock, where it is received and recorded. The act of reception constitutes the second event we consider. This event, R, is located at the rocket space origin, at the same location as the emission event E. Therefore, for the rocket observer, the space separation between event E and event R equals zero.

What is the time separation between events E and R in the rocket frame? The light travels 3 meters up to the mirror and 3 meters back down again, a total of 6 meters of distance. At the "standard" light speed of 1 meter of distance per meter of light-travel time, the flash takes a total of 6 meters of time to complete the round trip. In summary, for the rocket observer the event of reception, R, is separated from the event of emission, E, by zero meters in space and 6 meters in time.

What are the space and time separations of events E and R measured in the free-float laboratory frame? As measured in the laboratory, the rocket moves at high speed to the right (Figures 3-4 and 3-5). The rocket goes so fast that the simple

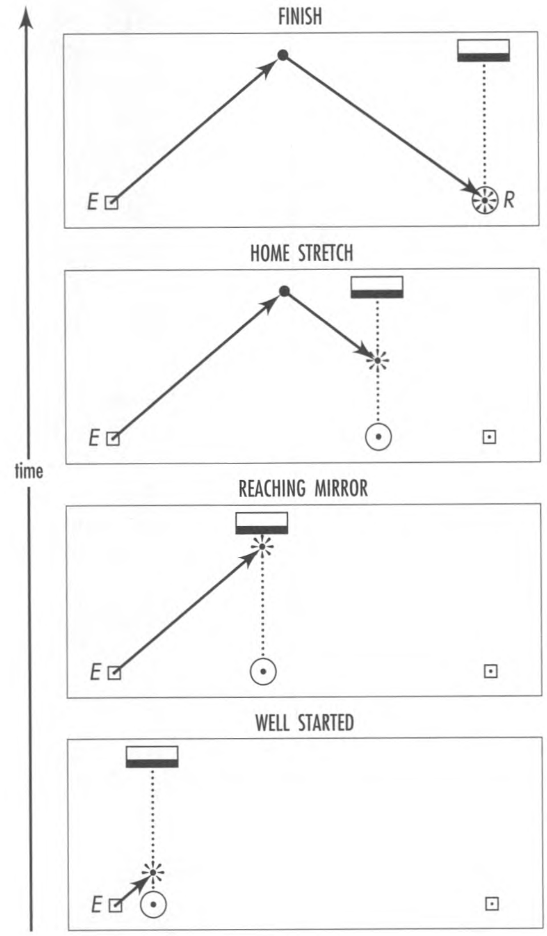


FIGURE 3-4. ***Plot of the flash path as recorded in the laboratory frame.*** *Time progresses from bottom to top:* ***Well started:*** *The flash (represented as an asterisk) has been emitted (event* E*) from a moving rocket clock (shown as a circle) that coincided with a laboratory clock (shown as a square).* ***Reaching mirror*** *and* ***Home stretch:*** *The flash reaches a mirror and reflects from it. The mirror moves along in step with the rocket clock.* ***Finish:*** *The flash is received (event* R*) back at the same rocket clock, which has moved in the laboratory frame to coincide with a second laboratory clock. Figure 3-5 shows the trajectory of the same flash in three different free-float frames.*

up-down track of the light in the rocket frame appears in the laboratory to have the profile of a tent, with its right-hand corner—the place of reception of the light—8 meters to the right of the starting point.

When does the event of reception, R, take place as registered in the laboratory frame? Note that it occurs at the time 6 meters in the rocket frame. All we know about everyday events urges us to say, "Why, obviously it occurs at 6 meters of time in the laboratory frame too." But no. More binding than preconceived expectations are the demands of the Principle of Relativity. Among those demands none ranks higher than this: The speed of light has the standard value 1 meter of distance in 1 meter of light-travel time in every free-float frame.

Greater distance of travel for light flash: longer time!

Figure 3-6 punches us in the eye with this point: The light flash travels *farther* as recorded in the laboratory frame than as recorded in the rocket frame. The perpendicular "altitude" of the mirror from the line along which the rocket reference clock moves has the same value in laboratory frame as in rocket frame no matter how fast the rocket—as shown in Section 3.6. Therefore on its slanted path toward and away from the mirror the flash must cover more distance in the laboratory frame than it does in the rocket frame. More distance covered means more time required at the "standard" light speed. We conclude that the time between events E and R is greater in the laboratory frame than in the rocket frame—a staggering result that stood physics on its ear when first proposed. There is no way out.

In the laboratory frame the flash has to go "up" 3 meters, as before, and "down" again 3 meters. But in addition it has to go 8 meters to the right: 4 meters to the right while rising to hit the mirror, and 4 meters more to the right while falling again to the receptor. The Pythagorean Theorem, applied to the right triangles of Figure 3-6, tells

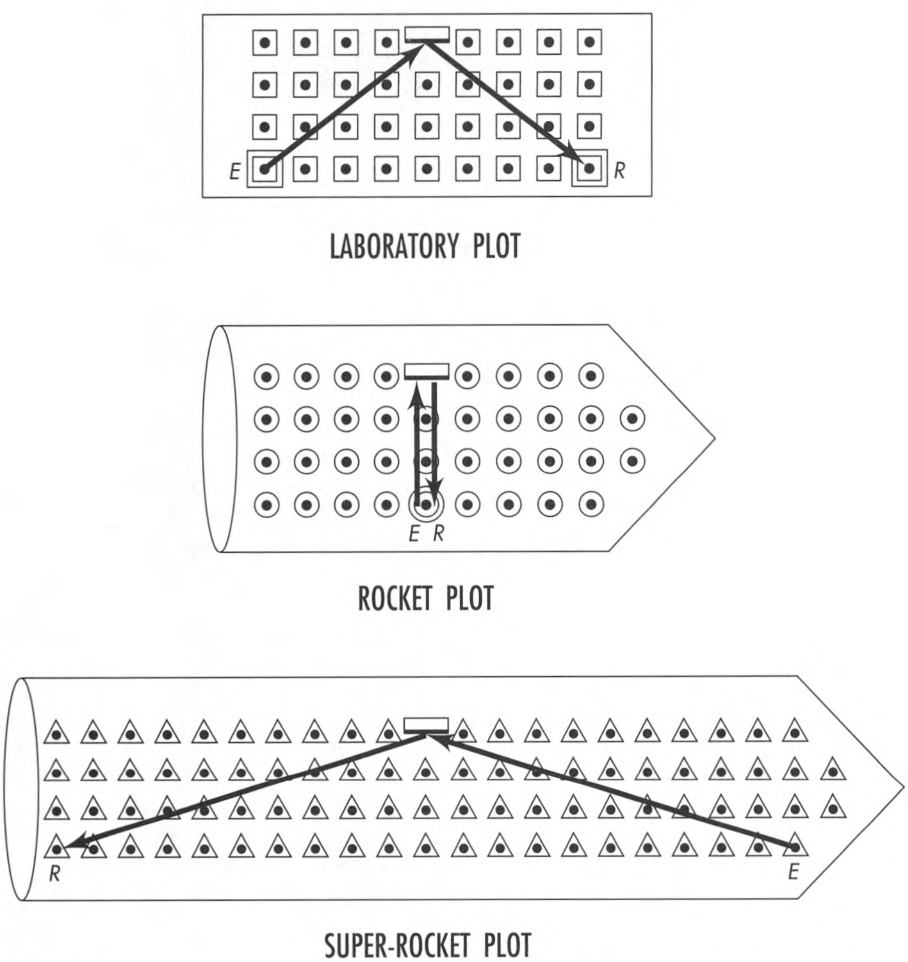
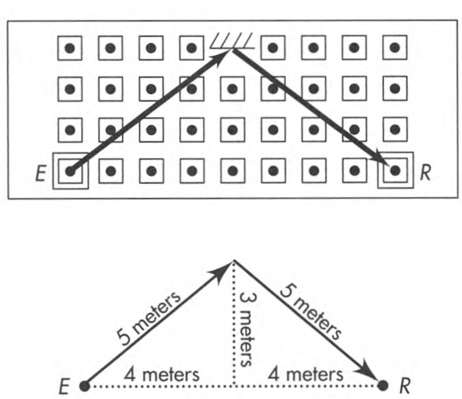


FIGURE 3-5. ***Plots of the path in space of a reflected flash of light as measured in three different frames, showing event* E*, emission of the flash, and event* R*, its reception after reflection.*** *Squares, circles, and triangles represent latticeworks of recording clocks in laboratory, rocket, and super-rocket frames, respectively. The super-rocket frame moves to the right with respect to the rocket, and with such relative speed that the event of reception,* R*, occurs to the left of the event of emission,* E*, as measured in the super-rocket frame. The reflecting mirror is fixed in the rocket, hence appears to move from left to right in the laboratory and from right to left in the super-rocket.*

FIGURE 3-6. ***Laboratory plot of the path of the light flash.*** *The flash rises 3 meters while it moves to the right 4 meters. Then it falls 3 meters as it moves an additional 4 meters to the right. From the Pythagorean Theorem, the total length of the flash path equals 5 meters plus 5 meters or 10 meters. Therefore 10 meters of light-travel time is the separation in time between emission event* E *and reception event* R *as measured in the laboratory frame.*



us that each slanted leg of the trip has length 5 meters:

$$(3 \text{ meters})^2 + (4 \text{ meters})^2 = (5 \text{ meters})^2$$

Thus the total length of the trip equals 10 meters, definitely longer than the length of the round trip, 6 meters, as observed in the rocket frame. Moreover, the light can cover that slanted and greater distance only at the standard rate of 1 meter of distance in 1 meter of light-travel time. Therefore there is no escape from saying that the time of reception as recorded in the laboratory frame equals 10 meters. Thus there is a great variance between what is recorded in the two frames (Figure 3-5, Laboratory plot and Rocket plot): separation in time and in space between the emission E of a pulse of light and its reception R after reflection.

In spite of the difference in space separation between events E and R and the difference in time lapse between these events as measured in laboratory and rocket frames, there exists a measure of their separation that has the same value for both observers. This is the interval calculated from the difference of squares of time and space separations (Table 3-1). For both observers the interval has the value 6 meters. The interval is an **invariant** between free-float frames.

Two central results are to be seen here, one of variance, the other of invariance. We discover first that typically there is not and cannot be an absolute time difference between two events. The difference in time depends on our choice of the free-float frame, which inertial frame we use to record events. There is no such thing as a simple concept of universal and absolute separation in time.

Between events: No absolute time, but invariant interval

Second, despite variance between the laboratory frame and the rocket frame in the values recorded for time and space separations individually, the difference between the squares of those separations is identical, that is, invariant with respect to choice of reference frame. The difference of squares obtained in this way defines the square of the interval. The invariant interval itself has the value 6 meters in this example.

TABLE 3-1

RECKONING THE SPACETIME INTERVAL FROM ROCKET AND LABORATORY MEASUREMENTS

	Rocket measurements		Laboratory measurements
Time from emission of the flash to its reception	6 meters	← DIFFERENT! →	10 meters
Distance from the point of emission of the flash to its point of reception	0 meters	← DIFFERENT! →	8 meters
Square of time	36 $(\text{meters})^2$		100 $(\text{meters})^2$
Square distance and subtract	-0 $(\text{meters})^2$		-64 $(\text{meters})^2$
Result of subtraction	36 $(\text{meters})^2$		36 $(\text{meters})^2$
This is the square of what measurement?	6 meters		6 meters

SAME SPACETIME
INTERVAL

3.8 INVARIANCE OF THE INTERVAL FOR *ALL* FREE-FLOAT FRAMES

super-rocket observer joins the agreement

The interval between two events has the same value for *all possible* relative speeds of overlapping free-float frames. As an example of this claim, consider a third free-float frame moving at a different speed with respect to the laboratory frame—a speed different from that of the rocket frame.

Super-rocket: Same interval between events

We now measure the same events of emission and reception from a "super-rocket frame" moving faster than the rocket (but not faster than light!) along the line between events E and R (Figure 3-5, Super-rocket plot). For convenience we arrange that the reference clock of this frame also coincides with reference clocks of the other two frames at event E.

Events E and R occur at the same place in the rocket frame. Between these two events the super-rocket moves to the *right* with respect to the rocket. As a result, the super-rocket observer records event R as occurring to the *left* of the emission event. How far to the left? That depends on the relative speed of the super-rocket frame.

The super-rocket is not super-size; rather it has super-speed. We adjust this super-speed so that the reception occurs 20 meters to the left of the emission for the super-rocket observer. Then the flash of light that rises vertically in the rocket must travel the same 3 meters upward in the super-rocket but also 10 meters to the left as it slants toward the mirror. Hence the distance it travels to the mirror in the super-rocket frame is the length of a hypotenuse, 10.44 meters:

$$\begin{aligned}(3 \text{ meters})^2 + (10 \text{ meters})^2 &= 9 \text{ meters}^2 + 100 \text{ meters}^2 = 109 \text{ meters}^2 \\ &= (10.44 \text{ meters})^2\end{aligned}$$

It must travel another 10.44 meters as it slants downward and leftward to the event of reception. The total distance traveled equals 20.88 meters. It follows that the total time lapse between E and R equals 20.88 meters of light-travel time for the super-rocket observer.

The speed of the super-rocket is very high. As a result the space separation between emission and reception is very great. But then the time separation is also very great. Moreover, the magnitude of the time separation is perfectly tailored to the size of the space separation. In consequence, the particular quantity equal to the difference of their squares has the value (6 meters)^2, no matter how great the space separation and time separation individually may be. For the super-rocket frame:

$$\begin{aligned}(20.88 \text{ meters})^2 - (20 \text{ meters})^2 &= 436 \text{ meters}^2 - 400 \text{ meters}^2 = 36 \text{ meters}^2 \\ &= (6 \text{ meters})^2\end{aligned}$$

In spite of the difference in space separation observed in the three frames (0 meters for the rocket, 8 meters for the laboratory, 20 meters for the super-rocket) and the difference in time separation (6 meters for the rocket, 10 meters for the laboratory, 20.88 meters for the super-rocket), the interval between the two events has the same value for all three observers:

$$\begin{aligned}
&\text{In general:} && (\text{time separation})^2 - (\text{space separation})^2 = (\text{interval})^2 \\
&\text{Rocket frame:} && (6 \text{ meters})^2 - (0 \text{ meters})^2 = (6 \text{ meters})^2 \\
&\text{Laboratory frame:} && (10 \text{ meters})^2 - (8 \text{ meters})^2 = (6 \text{ meters})^2 \\
&\text{Super-rocket frame:} && (20.88 \text{ meters})^2 - (20 \text{ meters})^2 = (6 \text{ meters})^2
\end{aligned}$$

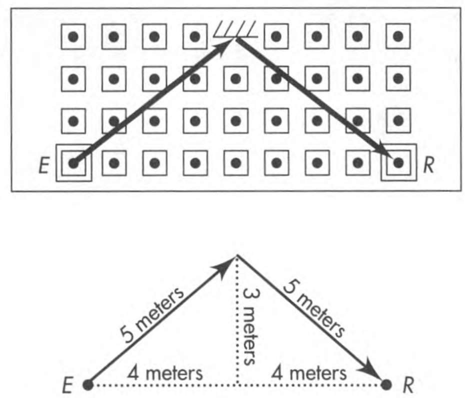


FIGURE 3-6 (repeated). *Laboratory plot of the path of the light flash.*

The laboratory observer clocks the time between the flash and its reception as 10 meters, in total disagreement with the 6 meters of timelike interval he figures between those two events. The observer in the super-rocket frame marks an even greater discrepancy, 20.88 meters of her time versus the 6 meters of timelike interval. Only for the rocket observer does clock time agree with interval. Why? Because only she sees reception at the same place as emission.

Invariance of interval from invariance of transverse dimension

The invariance of the interval can be seen at a glance in Figure 3-6. The hypotenuse of the first right triangle has a length equal to half the time separation between E and R. Its base has a length equal to half the space separation. To say that (time separation)^2 — (space separation)^2 has a standard value, and consequently to state that (half the time separation)^2 — (half the space separation)^2 has a standard value, is simply to say that the altitude of this right triangle has a fixed magnitude (3 meters in the diagram) for rocket and all super-rocket frames, no matter how fast they move. And this altitude has a length equal to half the interval between these two events.

SAMPLE PROBLEM 3-2

THE K^+ MESON

A beam of (unstable) K^+ mesons, traveling at a speed of $v = 0.868$, passes through two counters 9 meters apart. The particles suffer negligible loss of speed and energy in passing through the counters but give electrical pulses that can be counted. The first counter records 1000 pulses (1000 passing particles); the second records 250 counts (250 passing particles). This decrease arises almost entirely from decay of particles in flight. Determine the half-life of the K^+ meson in its own rest frame.

SOLUTION

Unstable particles of different kinds decay at different rates. By definition, the half-life of unstable particles of a particular species measures the particle wristwatch time during which — on the average — half of the particles decay. Half of the remaining particles decay in an additional time lapse equal to the same half-life, and so forth. In this case, one quarter of the K^+ particles remain after passage from counter to counter. Therefore the particles that survive experience the passage of two half-lives between counter and counter. We make the interval between those two passages, those two events, the center of our attention, because it has the same value in the laboratory frame where we do our measuring as it does in the free-float frame of the representative particle.

The keystone of the argument establishing the invariance of the interval between two events for all free-float frames? The Principle of Relativity, according to which there is no difference in the laws of physics between one free-float frame and another. This principle showed here in two very different ways. First, it said that distances at right angles to the direction of relative motion are recorded as of equal magnitude in the laboratory frame and the rocket frame (Section 3.6). Otherwise one frame could be distinguished from the other as the one with the shorter perpendicular distances.

Basis of invariance of interval: Principle of Relativity

Second, the Principle of Relativity demanded that the speed of light be the same in the laboratory frame as in the rocket frame. The speed being the same, the fact that the light-travel path in the laboratory frame (the hypotenuse of two triangles) is longer than the simple round-trip path in the rocket frame (the altitudes of these two triangles: up 3 meters and down again) directly implies a longer time in the laboratory frame than in the rocket frame.

In brief, one elementary triangle in Figure 3-6 displays four great ideas that underlie all of special relativity: invariance of perpendicular distance, invariance of the speed of light, dependence of space and time separations upon the frame of reference, and invariance of the interval.

3.9 SUMMARY

same laws for all; invariant interval for all

The **Principle of Relativity** says that the laws of physics are the same in every inertial (free-float) reference frame (Section 3.1). This simple principle has important consequences. Specifically:

$$(\text{interval})^2 = \begin{pmatrix}\text{separation}\\ \text{in lab}\\ \text{time}\end{pmatrix}^2 - \begin{pmatrix}\text{separation}\\ \text{in lab}\\ \text{position}\end{pmatrix}^2 = \begin{pmatrix}\text{separation}\\ \text{in moving-}\\ \text{particle time}\end{pmatrix}^2 - \begin{pmatrix}\text{separation}\\ \text{in moving-}\\ \text{particle position}\end{pmatrix}^2$$

$$= \left(\frac{9 \text{ meters of distance}}{\begin{matrix}0.868 \text{ meters of distance}\\ \text{per meter of time}\end{matrix}}\right)^2 - \begin{pmatrix}9 \text{ meters}\\ \text{of distance}\end{pmatrix}^2 = (2 \text{ half-lives})^2 - \begin{pmatrix}\text{zero separation}\\ \text{in space (in}\\ \text{particle frame)}\\ \text{between those}\\ \text{two events}\end{pmatrix}^2$$

$$= \begin{pmatrix}10.368 \text{ meters}\\ \text{of light-travel time}\end{pmatrix}^2 - \begin{pmatrix}9 \text{ meters}\\ \text{of distance}\end{pmatrix}^2 = (2 \text{ half-lives})^2$$

A little arithmetic tells us that two half-lives total 5.15 meters of light-travel time. Consequently the K^+ half-life itself is 2.57 meters of time or (2.57 meters)/(3.00×10^8 meters/second) $= 8.5 \times 10^{-9}$ second or 8.5 nanoseconds.

1. Two events that lie along the direction of relative motion between two frames cannot be simultaneous as measured in both frames **(relativity of simultaneity).** (Section 3.4)
2. An object in high-speed motion is measured to be shorter along its direction of motion than its **proper length,** measured in its rest frame **(Lorentz contraction).** (Section 3.5)
3. The dimensions of moving objects transverse to their direction of relative motion are measured to be the same, whatever the relative speed **(invariance of transverse distances).** (Section 3.6)
4. Two events with separation only transverse to the direction of relative motion and simultaneous in either frame are simultaneous in both. (Section 3.6)

BOX 3-3

FASTER THAN LIGHT?

We always want to go faster. Faster than what? Faster than anything has gone before. What is our greatest possible speed, according to the theory of relativity? The speed of light in a vacuum! How do we know that this is the greatest possible speed that we can travel? Many lines of evidence reach this conclusion. Rocket speed greater than the speed of light would lead to the destruction of the essential relation between cause and effect, a result explored in Special Topic: Lorentz Transformation (especially Box L-1) and in Chapter 6. In particular, we could find a frame in which a faster-than-light object arrives before it starts! Moreover, in particle accelerators built over several decades we have spent hundreds of millions of dollars effectively trying to accelerate electrons and protons to the greatest possible speed—which by experiment never exceeds light speed.

The conclusion that no thing can move faster than light arises also from the invariance of the interval. To see this, let a rocket emit two flashes of light a time t' apart as measured in the rocket frame. (Use a prime to distinguish rocket measurements from laboratory measurements.) In the rocket frame the two emissions occur at the same place: the separation x' between them equals zero. Let t and x be the corresponding separations in time and space as measured in the laboratory frame. Then the invariance of the interval tells us that the three quantities t', t, and x are related by the equation

$$(t')^2 - (x')^2 = (t')^2 - (0)^2 = t^2 - x^2$$

whence

$$(t')^2 = t^2 - x^2 \tag{3-1}$$

In the laboratory frame the rocket is moving with some speed; give this speed the symbol v. The distance x between emissions is just the distance that the rocket moves in time t in the laboratory frame. The relation between

5. The spacetime interval between two events is invariant — it has the same value in laboratory and rocket frames (Sections 3.7 and 3.8):

$$(\text{interval})^2 = \overset{\text{Laboratory}}{\begin{pmatrix}\text{time}\\ \text{separation}\end{pmatrix}^2} - \overset{\text{Laboratory}}{\begin{pmatrix}\text{space}\\ \text{separation}\end{pmatrix}^2}$$

$$= \overset{\text{Rocket}}{\begin{pmatrix}\text{time}\\ \text{separation}\end{pmatrix}^2} - \overset{\text{Rocket}}{\begin{pmatrix}\text{space}\\ \text{separation}\end{pmatrix}^2}$$

6. In any free-float frame, no object moves with a speed greater than the speed of light (Box 3-3).

distance, time, and speed is

$$x = vt \tag{3-2}$$

Substitute this into equation (3-1) to obtain $(t')^2 = t^2 - (vt)^2 = t^2\,[1 - v^2]$, or

$$t' = t\,(1 - v^2\,)^{1/2} \tag{3-3}$$

Now, v is the speed of the rocket. How large can that speed be? Equation (3-3) makes sense for any rocket speed less than the speed of light, or when v has a value less than one.

Suppose we try to force the rocket to move faster than the speed of light. If we should succeed, v would have a value greater than one. Then v^2 also would have a value greater than one. But in this case the expression $1 - v^2$ would have a negative value and its square root would have no physical meaning. In a formal mathematical sense, the rocket time t' would be an imaginary number for the case of rocket speed greater than the speed of light. But clocks do not read imaginary time; they read real time — three hours, for example. Therefore a rocket speed greater than the speed of light leads to an impossible consequence.

Equation (3-3) does not forbid a rocket to go as close to the speed of light as we wish, as long as this speed remains less than the speed of light. For v very close to the speed of light, equation (3-3) tells us that the rocket time can be very much smaller than the laboratory time. Now suppose that emission of the first flash occurs when the rocket passes Earth on its outward trip to a distant star. Let emission of the second flash occur as the rocket *arrives* at that distant star. No matter how long the laboratory time t between these two events, we can find a rocket speed, v, such that the rocket time t' is as small as we wish. This means that in principle we can go to any remote star in as short a rocket time as we want. In brief, although our speed is limited to less than the speed of light, the distance we can travel in a lifetime has no limitation. We can go anywhere! This result is explored further in Chapter 4.

BOX 3-4

DOES A MOVING CLOCK *REALLY* "RUN SLOW"?

You keep saying, "The time between clock-ticks is shorter as MEASURED in the rest frame of the clock than as MEASURED in a frame in which the clock is moving." I am interested in reality, not someone's measurements. Tell me what really happens!

What is reality? You will have your own opinion and speculations. Here we pose two related scientific questions whose answers may help you in forming your opinion.

Are differences in clock rates *really* verified by experiment?
Different values of the time between two events as observed in different frames? Absolutely! Energetic particles slam into solid targets in accelerators all over the world, spraying forward newly created particles, some of which decay in very short times as measured in their rest frames. But these "short-lived" particles survive much longer in the laboratory frame as they streak from target to detector. In consequence, the detector receives a much larger fraction of the undecayed fast-moving particles than would be predicted from their decay times measured at rest. This result has been tested thousands of times with many different kinds of particles. Such experiments carried out over decades lead to dependable, consistent, repeatable results. As far as we can tell, they are correct, true, and reliable and cannot effectively be denied. If that is what you personally mean by "real," then these results are "what really happens."

Does something about a clock *really* change when it moves, resulting in the observed change in tick rate?
Absolutely not! Here is why: Whether a free-float clock is at rest or in motion in the frame of the observer is controlled by the observer. You want the clock

REFERENCES

Introductory quote: A. Sommerfeld, *Naturwissenschaftliche Rundschau,* Volume 1, pages 97–100, reprinted in *Gesammelte Schriften* (Vieweg, Braunschweig, 1968), Volume IV, pages 640–643.

Galileo quote, Section 3.1: Galileo Galilei, *Dialogue Concerning the Two Chief World Systems — Ptolemaic and Copernican,* first published February 1632; the translation quoted here is by Stillman Drake (University of California Press, Berkeley, 1962), pages 186ff. Galileo's writings, along with those of Dante, by reason of their strength and aptness, are treasures of human thought, studied today in Italy by secondary school students as part of a great literary heritage.

Einstein quote, Box 3-1: Albert Einstein, "On the Electrodynamics of Moving Bodies," *Annalen der Physik,* Volume 17, pages 891–921 (1905), translated by Arthur I. Miller in *Albert Einstein's Special Theory of Relativity* (Addison-Wesley, Reading, Mass., 1981), page 392.

to be at rest? Move along with it! Now do you want the clock to move? Simply change your own velocity! This is true even when you and the clock are separated by the diameter of the solar system. The magnitude of the clock's steady velocity is entirely under your control. Therefore the time between its ticks as measured in your frame is determined by your actions. How can your change of motion affect the inner mechanism of a distant clock? It cannot and does not.

Every time you change your motion on Earth — and even when you sit down, letting the direction of your velocity change as Earth rotates — you change the rate at which the planets revolve around Sun, as measured in your frame. (You also change the shape of planetary orbits, contracting them along the direction of your motion relative to Sun.) Do you think this change on your velocity really affects the workings of the "clock" we call the solar system? If so, what about a person who sits down on the other side of Earth? That person moves in the opposite direction around the center of Earth, so the results are different from yours. Are each of you having a different effect on the solar system? And are there still different effects — different solar-system clocks — for observers who could in principle be scattered on other planets?

We conclude that free-float motion does not affect the structure or operation of clocks (or rods). If this is what you mean by reality, then there are *really* no such changes due to uniform motion.

Is there some unity behind these conflicting measurements of time and space? Yes! The interval: the proper time (wristwatch time) between ticks of a clock as measured in a frame in which ticks occur at the same place, in which the clock is at rest. Proper time can also be calculated by all free-float observers, whatever their state of motion, and all agree on its value. Behind the confusing clutter of conflicting measurements stands the simple, consistent, powerful view provided by spacetime.

ACKNOWLEDGMENTS

The idea for Box 3-1 was suggested by Kenneth L. Laws. Box 3-4 and the argument for Section 3.6, Invariance of Transverse Dimension, is adapted from material by William A. Shurcliff, private communications. Sample Problem 3-2 is adapted from A. P. French, *Special Relativity* (W. W. Norton, New York, 1968), page 121.

CHAPTER 3 EXERCISES

PRACTICE

3-1 relativity and swimming

The idea here is to illustrate how remarkable is the invariance of the speed of light (light speed same in all free-float frames) by contrasting it with the case of a swimmer making her way through water.

Light goes through space at 3×10^8 meters/second, and the swimmer goes through the water at 1 meter/second. "But how can there otherwise be any difference?" one at first asks oneself.

For a light flash to go down the length of a 30-meter spaceship and back again takes

$$\begin{aligned} \text{time} &= (\text{distance})/(\text{speed}) \\ &= 2 \times (30 \text{ meters})/(3 \times 10^8 \text{ meters/second}) \\ &= 2 \times 10^{-7} \text{ second} \end{aligned}$$

as measured in the spaceship, regardless of whether the ship is stationary at the spaceport or is zooming past it at high speed.

Check how very different the story is for the swimmer plowing along at 1 meter/second with respect to the water.

a How long does it take her to swim down the length of a 30-meter pool and back again?

b How long does it take her to swim from float A to float B and back again when the two floats, A and B, are still 30 meters apart, but now are being towed through a lake at 1/3 meter/second? **Discussion:** When the swimmer is swimming in the same direction in which the floats are being towed, what is her speed relative to the floats? And how great is the distance she has to travel expressed in the "frame of reference" of the floats? So how long does it take to travel that leg of her trip? Then consider the same three questions for the return trip.

c Is it true that the total time from A to B and back again is independent of the reference system ("stationary" pool ends vs. moving floats)?

d Express in the cleanest, clearest, sharpest one-sentence formulation you can the difference between what happens for the swimmer and what happens for a light flash.

3-2 Einstein puzzler

When Albert Einstein was a boy of 16, he mulled over the following puzzler: A runner looks at herself in a mirror that she holds at arm's length in front of her. If she runs with nearly the speed of light, will she be able to see herself in the mirror? Analyze this question using the Principle of Relativity.

3-3 construction of clocks

For the measurement of time, we have made no distinction among spring clocks, quartz crystal clocks, biological clocks (aging), atomic clocks, radioactive clocks, and a clock in which the ticking element is a pulse of light bouncing back and forth between two mirrors (Figure 1-3). Let all these clocks be adjusted by the laboratory observer to run at the same rate when at rest in the laboratory. Now let the clocks all be accelerated gently to a high speed in a rocket, which then turns off its engines. Make a simple but powerful argument that the free-float rocket observer will also measure these different clocks all to run at the same rate as one another. Does it follow that the (common) clock rate of these clocks measured by the rocket observer is the same as their (common) rate measured by the laboratory observer as they pass by in the rocket?

3-4 the Principle of Relativity

Two overlapping free-float frames are in uniform relative motion. On the following list, mark with a "yes" the quantities that must *necessarily* be the same as measured in the two frames. Mark with a "no" the quantities that are *not* necessarily the same as measured in the two frames.

a time it takes for light to go one meter of distance in a vacuum

b spacetime interval between two events

c kinetic energy of an electron

d value of the mass of the electron

e value of the magnetic field at a given point

f distance between two events

g structure of the DNA molecule

h time rate of change of momentum of a neutron

3-5 many unpowered rockets

In the laboratory frame, event 1 occurs at $x = 0$ light-years, $t = 0$ years. Event 2 occurs at $x = 6$ light-years, $t = 10$ years. In all rocket frames, event 1 also occurs at the position 0 light-years and the time 0 years. The y- and z-coordinates of both events are zero in both frames.

a In rocket frame A, event 2 occurs at time $t' = 14$ years. At what position x' will event 2 occur in this frame?

b In rocket frame B, event 2 occurs at position $x'' = 5$ light-years. At what time t'' will event 2 occur in this frame?

c How fast must rocket frame C move if events 1 and 2 occur at the same place in this rocket frame?

d What is the time between events 1 and 2 in rocket frame C of part c?

3-6 down with relativity!

Mr. Van Dam is an intelligent and reasonable man with a knowledge of high school physics. He has the following objections to the theory of relativity. Answer each of Mr. Van Dam's objections decisively—without criticizing him. If you wish, you may present a single connected account of how and why one is driven to relativity, in which these objections are all answered.

a "Observer A says that B's clock goes slow, and observer B says that A's clock goes slow. This is a logical contradiction. Therefore relativity should be abandoned."

b "Observer A says that B's meter sticks are contracted along their direction of relative motion, and observer B says that A's meter sticks are contracted. This is a logical contradiction. Therefore relativity should be abandoned."

c "Relativity does not even have a unique way to *define* space and time coordinates for the instantaneous position of an object. Laboratory and rocket observers typically record different coordinates for this position and time. Therefore anything relativity says about the velocity of the object (and hence about its motion) is without meaning."

d "Relativity postulates that light travels with a standard speed regardless of the free-float frame from which its progress is measured. This postulate is certainly wrong. Anybody with common sense knows that travel at high speed in the direction of a receding light pulse will decrease the speed with which the pulse recedes. Hence a flash of light *cannot* have the same speed for observers in relative motion. With this disproof of the basic postulate, all of relativity collapses."

e "There isn't a single experimental test of the *results* of special relativity."

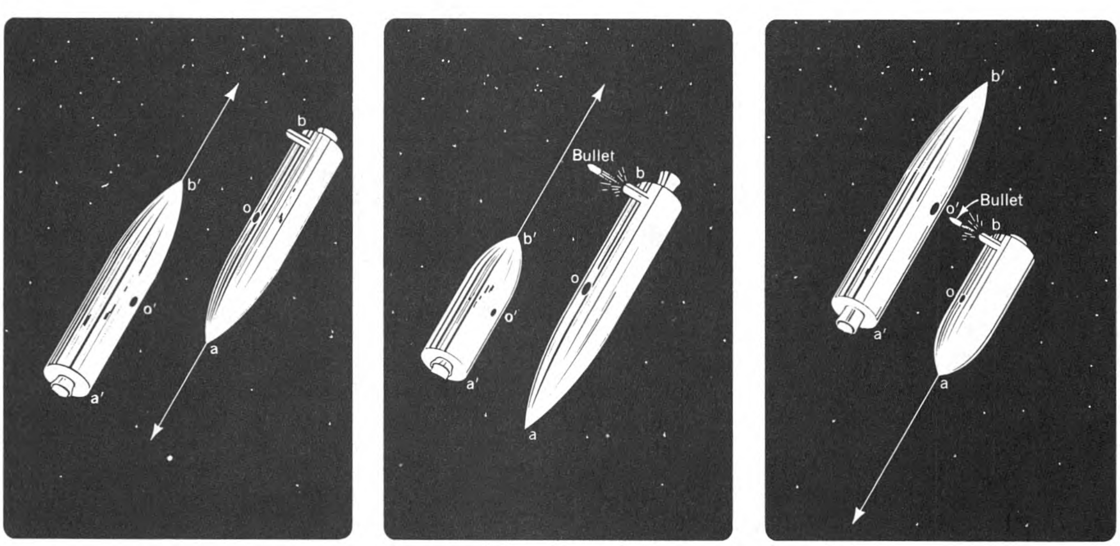
f "Relativity offers no way to describe an event without coordinates—and no way to speak about coordinates without referring to one or another particular reference frame. However, physical events have an existence independent of all choice of coordinates and all choice of reference frame. Hence relativity—with its coordinates and reference frames—cannot provide a valid description of these events."

g "Relativity is preoccupied with how we *observe* things, not what is *really* happening. Hence it is not a scientific theory, since science deals with reality."

PROBLEMS

3-7 space war

Two rockets of equal rest length are passing "head on" at relativistic speeds, as shown in the figure (left). Observer o has a gun in the tail of her rocket pointing perpendicular to the direction of relative motion



EXERCISE 3-7. ***Left:*** *Two rocket ships passing at high speed.* ***Center:*** *In the frame of* o *one expects a bullet fired when* a *coincides with* a′ *to miss the other ship.* ***Right:*** *In the frame of* o′ *one expects a bullet fired when* a *coincides with* a′ *to hit the other ship.*

(center). She fires the gun when points a and a' coincide. In her frame the other rocket ship is Lorentz contracted. Therefore o expects her bullet to miss the other rocket. But in the frame of the other observer o' it is the rocket ship of o that is measured to be Lorentz contracted (right). Therefore when points a and a' coincide, observer o' should observe a hit.

Does the bullet actually hit or miss? Pinpoint the looseness of the language used to state the problem and the error in one figure. Show that your argument is consistent with the results of the Train Paradox (Section 3.4).

3-8 Čerenkov radiation

No particle has been observed to travel faster than the speed of light in a *vacuum*. However particles have been observed that travel in a material medium faster than the speed of light *in that medium*. When a charged particle moves through a medium faster than light moves in that medium, it radiates coherent light in a cone whose axis lies along the path of the particle. (Note the rough similarity to waves created by a motorboat speeding across calm water and the more exact similarity to the "cone of sonic boom" created by a supersonic aircraft.) This is called Čerenkov radiation (Russian $\check{C}$ is pronounced as "ch"). Let v be the speed of the particle in the medium and v_{light} be the speed of light in the medium.

a From this information use the first figure to show that the half-angle ϕ, of the light cone is given by the expression

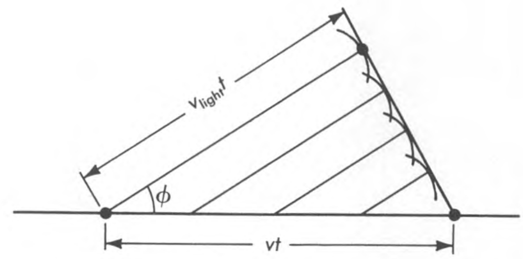
$$\cos \phi = v_{light}/v$$

b Consider the plastic with the trade name Lucite, for which $v_{light} = 2/3$. What is the minimum velocity that a charged particle can have if it is to produce Čerenkov radiation in Lucite? What is the *maximum* angle ϕ at which Čerenkov radiation can be produced in Lucite? Measurement of the angle provides a good way to measure the velocity of the particle.

c In water the speed of light is approximately $v_{light} = 0.75$. Answer the questions of part **b** for the case of water. See the second figure for an application of Čerenkov radiation in water.

3-9 aberration of starlight

A star lies in a direction generally perpendicular to Earth's direction of motion around Sun. Because of Earth's motion, the star appears to an Earth observer to lie in a slightly different direction than it would



EXERCISE 3-8, first figure. *Calculation of Čerenkov angle ϕ.*

EXERCISE 3-8, second figure. *Use of Čerenkov radiation for indirect detection of neutrinos in the Deep Underwater Muon and Neutrino Detector (DUMAND) 30 kilometers off Keahole Point on the island of Hawaii. Neutrinos have no electric charge and their mass, if any, has so far escaped detection (Box 8-1). Neutrinos interact extremely weakly with matter, passing through Earth with almost no collisions. Indeed, the DUMAND detector array selects for analysis only neutrinos that come upward through Earth. In this way Earth itself acts as a shield to eliminate all other cosmic-ray particles.*

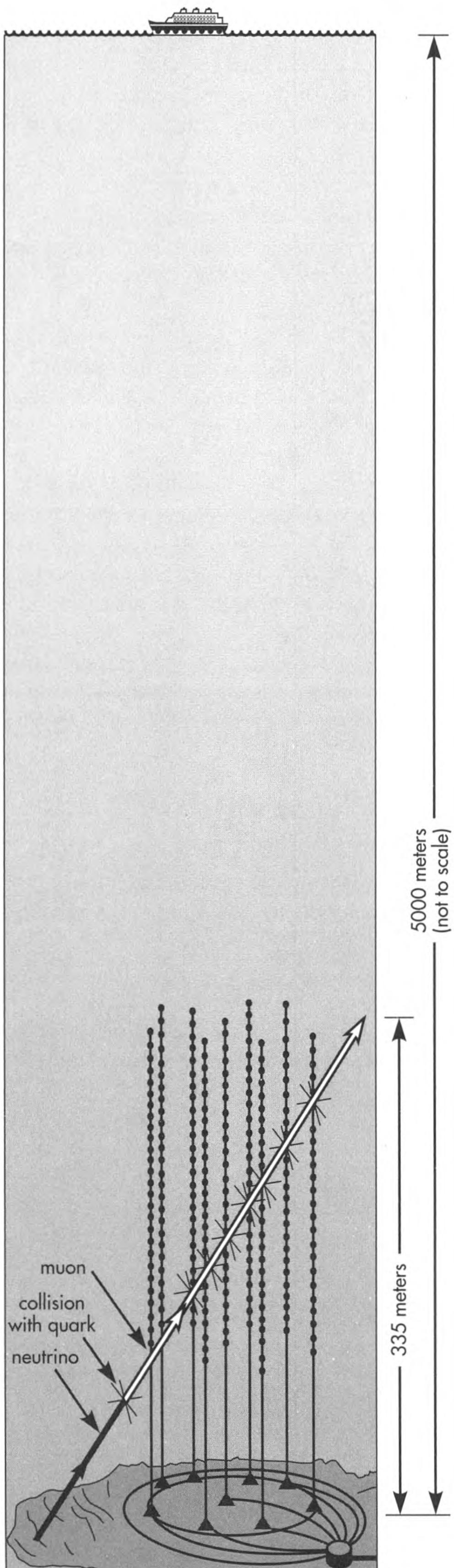
What are possible sources for these neutrinos? Theory predicts the emission of very high-energy (greater than 10^{12} electron-volt) neutrinos from matter plunging toward a black hole. Black holes may be the energy sources for extra-bright galactic nuclei and for quasars — small, distant, enigmatic objects shining with the light of hundreds of galaxies (Section 9.8). Information about conditions deep within these astronomical structures may be carried by neutrinos as they pierce Earth and travel upward through the DUMAND detector array.

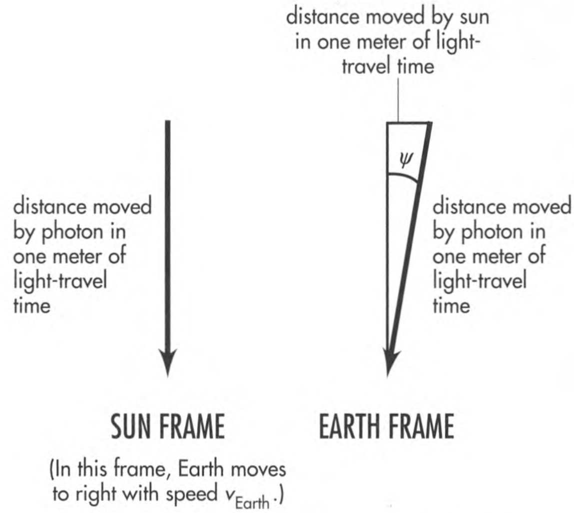
In a rare event, a neutrino moving through the ocean slams into one of the quarks that make up a proton or a neutron in, say, an oxygen nucleus in the water, creating a burst of particles. All of these particles are quickly absorbed by the surrounding water except a stable negatively charged muon, 207 times the mass of the electron (thus sometimes called a "fat electron"). This muon streaks through the water in the same direction as the neutrino that created it and at a speed greater than that of light in water, thus emitting Čerenkov radiation. The Čerenkov radiation is detected by photomultiplier tubes in an array anchored to the ocean floor.

Photomultipliers are strung along 9 vertical cables, 8 cables spaced around a circle 100 meters in diameter on the ocean floor, the ninth cable rising from the center of the circle. Each cable is 335 meters long and holds 24 glass spheres positioned 10 meters apart on the top 230 meters of its length. There are no detectors on the bottom 110 meters, in order to avoid any cloud of sediments from the bottom. Above the bottom, the water is so clear and modern photodetectors so sensitive that Čerenkov radiation can be detected from a muon that passes within 40 meters of a detector.

Photomultipliers in the glass spheres detect Čerenkov radiation from the passing muons, transmitting this signal through underwater optical fibers to computers on the nearby island of Hawaii. The computers select for examination only those events in which (1) several optical sensors detect bursts that are (2) within 40 meters or so of a straight line, (3) spaced in time to show that the particle is moving at essentially the speed of light in a vacuum, and (4) from a particle moving upward through the water. A system of sonar beacons and hydrophones tracks the locations of the photomultipliers as the strings sway with the slow ocean currents. As a result, the direction of motion of the original neutrino can be recorded to an accuracy of one degree.

The DUMAND facility is designed to create a new sky map of neutrino sources to supplement our knowledge of the heavens, so far obtained primarily from the electromagnetic spectrum (radio, infrared, optical, ultraviolet, X-ray, gamma ray).





EXERCISE 3-9. *Aberration of starlight. Not to scale.*

appear to an observer at rest relative to Sun. This effect is called **aberration.** Using the diagram, find this apparent difference of direction.

a Find a trigonometric expression for the aberration angle ψ shown in the figure.

b Evaluate your expression using the speed of Earth around Sun, $v_{\text{Earth conv}} = 30$ kilometers/second. Find the answer in radians and in seconds of arc. (One degree equals 60 minutes of arc; one minute equals 60 seconds of arc.) This change in apparent position can be detected with sensitive equipment.

c The nonrelativistic answer to this problem—the answer using nonrelativistic physics—is $\tan \psi = v_{\text{Earth}}$ (in meters/meter). Do you think that the experimental difference between relativistic and nonrelativistic answers for stellar aberration observed from Earth can be the basis of a crucial experiment to decide between the correctness of the two theories?

Discussion: Of course we cannot climb off Earth and view the star from the Sun frame. But Earth reverses direction every six months (with respect to what?), so light from a "transverse star" viewed in, say, July will appear to be shifted through twice the aberration angle calculated in part **b** compared with the light from the same star in January. New question: Since the background of stars behind the one under observation also shifts due to aberration, how can the effect be measured at all?

d A rocket in orbit around Earth suddenly changes its velocity from a very small fraction of the speed of light to $v = 0.5$ with respect to Sun, moving in the same direction as Earth is moving around Sun. In what direction will the rocket astronaut now see the star of parts **a** and **b**?

3-10 the expanding universe

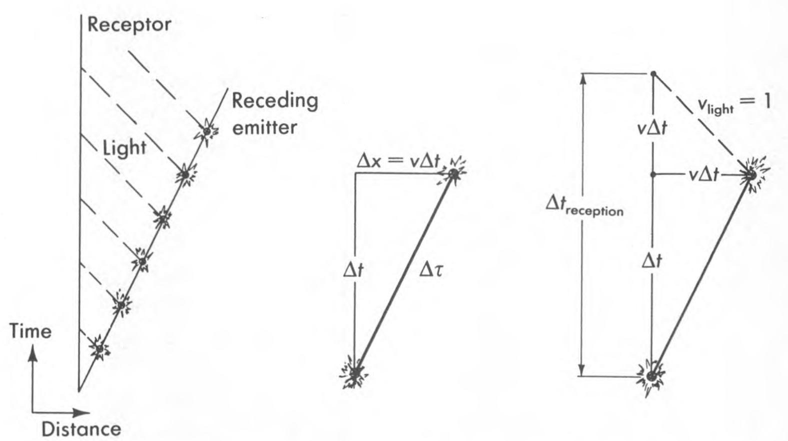
a A giant bomb explodes in otherwise empty space. What is the nature of the motion of one fragment relative to another? And how can this relative motion be detected? **Discussion:** Imagine each fragment equipped with a beacon that gives off flashes of light at regular, known intervals $\Delta\tau$ of time as measured in its own frame of reference (proper time!). Knowing this interval between flashes, what method of detection can an observer on one fragment employ to determine the velocity v—relative to her—of any other fragment? Assume that she uses, in making this determination, (1) the known proper time $\Delta\tau$ between flashes and (2) the time $\Delta t_{reception}$ between the arrival of consecutive flashes at her position. (This is *not* equal to the time Δt in her frame between the emission of the two flashes from the receding emitter; see the figure.) Derive a formula for v in terms of proper time lapse $\Delta\tau$ and $\Delta t_{reception}$. How will the measured recession velocity depend on the distance from one's own fragment to the fragment at which one is looking? Hint: In any given time in any given frame, fragments evidently travel distances in that frame from the point of explosion that are in direct proportion to their velocities in that frame.

b How can observation of the light from stars be used to verify that the universe is expanding? **Discussion:** Atoms in hot stars give off light of different frequencies characteristic of these atoms ("spectral lines"). The observed period of the light in each spectral line from starlight can be measured on Earth. From the pattern of spectral lines the kind of atom emitting the light can be identified. The same kind of atom can then be excited in the laboratory to emit light while at rest and the proper period of the light in any spectral line can be measured. Use the results of part **a** to describe how the observed period of light in one spectral line from starlight can be compared to the proper period of light in the same spectral line from atoms at rest in the laboratory to give the velocity of recession of the star that emits the light. This observed change in period due to the velocity of the source is called the Doppler shift. (For a more detailed treatment of Doppler shift, see the exercises for Chapters 5 and 8.) If the universe began in a gigantic explosion, how must the observed velocities of recession of different stars at different distances compare with one another? Slowing down during expansion—by gravitational attraction or otherwise—is to be neglected here but is considered in more complete treatments.

c The brightest steadily shining objects in the heavens are called quasars, which stands for "quasi-stellar objects." A single quasar emits more than 100 times the light of our entire galaxy. One possible source of quasar energy is the gravitational energy released as material falls into a black hole (Section 9.8). Because they are so bright, quasars can be observed at great distances. As of 1991, the greatest observed quasar red shift $\Delta t_{reception}/\Delta\tau$ has the value 5.9. According to the theory of this exercise, what is the velocity of recession of this quasar, as a fraction of the speed of light?

3-11 law of addition of velocities

In a spacebus a bullet shoots forward with speed 3/4 that of light as measured by travelers in the bus. The spacebus moves forward with speed 3/4 light speed as measured by Earth observers. How fast does the bullet move as measured by Earth observers: $3/4 + 3/4 = 6/4 = 1.5$ times the speed of light? No! Why not? Because (1) special relativity predicts that noth-



EXERCISE 3-10. *Calculation of the time $t_{reception}$ between arrival at observer of consecutive flashes from receding emitter. Light moves one meter of distance in one meter of time, so lines showing motion of light are tilted at $\pm 45°$ from the vertical.*

ing can travel faster than light, and (2) hundreds of millions of dollars have been spent accelerating particles ("bullets") to the fastest possible speed without anyone detecting a single particle that moves faster than light in a vacuum. Then where is the flaw in our addition of velocities? And what is the correct law of addition of velocities? These questions are answered in this exercise.

a First use Earth observers to record the motions of the spacebus (length L measured in the Earth frame, speed v_{rel}) and the streaking bullet (speed v_{bullet}). The bullet starts at the back of the bus. To give it some competition, let a light flash (speed = 1) race the bullet from the back of the bus toward the front. The light flash wins, of course, reaching the front of the bus in time $t_{forward}$. And $t_{forward}$ is also equal to the distance that the light travels in this time. Show that this distance (measured in the Earth frame) equals the length of the bus plus the distance the bus travels in the same time:

$$t_{forward} = L + v_{rel}\, t_{forward} \text{ or } t_{forward} = \frac{L}{1 - v_{rel}} \tag{1}$$

b In order to rub in its advantage over the bullet, the light flash reflects from the front of the bus and moves backward until, after an additional time $t_{backward}$, it rejoins the forward-plodding bullet. This meeting takes place next to the seat occupied by Fred, who sits a distance fL behind the front of the bus, where f is a fraction of the bus length L. Show that for this leg of the trip the Earth-measured distance $t_{backward}$ traveled by the light flash can also be expressed as

$$t_{backward} = fL - v_{rel}\, t_{backward} \quad \text{or} \quad t_{backward} = \frac{fL}{1 + v_{rel}} \tag{2}$$

c The light flash has moved forward and then backward with respect to Earth. What is the *net* forward distance covered by the light flash at the instant it rejoins the bullet? Equate this with the forward distance moved by the bullet (at speed v_{bullet}) to obtain the equation

$$v_{bullet}(t_{forward} + t_{backward}) = t_{forward} - t_{backward}$$

or

$$(1 + v_{bullet})\, t_{backward} = (1 - v_{bullet})\, t_{forward} \tag{3}$$

d What are we after? We want a relation between the bullet speed v_{bullet} as measured in the Earth frame and the bullet speed, call it v'_{bullet} (with a prime), as measured in the spacebus frame. The times given in parts **a**, **b**, and **c** are of no use to this end. Worse, we already know that times between events are typically different as measured in the spacebus frame than times between the same events measured in the Earth frame. So get rid of these times! Moreover, the Lorentz-contracted length L of the spacebus itself as measured in the Earth frame will be different from its rest length measured in the bus frame (Section 3.5). So get rid of L as well. Equations (1), (2), and (3) can be treated as three equations in the three unknowns $t_{forward}$, $t_{backward}$, and L. Substitute equations for the times (1) and (2) into equation (3). Lucky us: The symbol L cancels out of the result. Show that this result can be written

$$f = \frac{(1 - v_{bullet})}{(1 + v_{bullet})}\frac{(1 + v_{rel})}{(1 - v_{rel})} \tag{4}$$

e Now repeat the development of parts **a** through **d** for the spacebus frame, with respect to which the spacebus has its rest length L' and the bullet has speed v'_{bullet} (both with primes). Show that the result is:

$$f = \frac{(1 - v'_{bullet})}{(1 + v'_{bullet})} \tag{5}$$

Discussion: Instead of working hard, work smart! Why not use the old equations (1) through (4) for the spacebus frame? Because there is no relative velocity v_{rel} in the spacebus frame; the spacebus is at rest in its own frame! No problem: Set $v_{rel} = 0$ in equation (4), replace v_{bullet} by v'_{bullet} and obtain equation (5) directly from equation (4). If this is too big a step, carry out the derivation from the beginning in the spacebus frame.

f Do the two fractions f in equations (4) and (5) have the same value? In equation (4) the number f locates Fred's seat in the bus as a fraction of the total length of the bus in the Earth frame. In equation (5) the number f locates Fred's seat in the bus as a fraction of the total length of the bus in the bus frame. But this fraction must be the same: Fred cannot be halfway back in the Earth frame and, say, three quarters of the way back in the spacebus frame. Equate the two expressions for f given in equations (4) and (5) and solve for v_{bullet} to obtain the Law of Addition of Velocities:

$$v_{bullet} = \frac{v'_{bullet} + v_{rel}}{1 + v'_{bullet}\, v_{rel}} \tag{6}$$

g Explore some consequences of the Law of Addition of Velocities.

(1) An express bus on Earth moves at 108 kilometers/hour (approximately 67 miles/hour or 30 meters per second). A bullet moves forward with speed 600 meters/second with respect to the bus. What are the values of v_{rel} and v'_{bullet} in meters/meter? What is the value of their product in the denominator of equation (6)? Does this product of speeds increase the value of the denominator significantly over the value unity? Therefore what approximate form does equation (6) take for everyday speeds? Is this the form you would expect from your experience?

(2) Analyze the example that began this exercise: Speed of bullet with respect to spacebus $v'_{\text{bullet}} = 3/4$; speed of spacebus with respect to Earth $v_{\text{rel}} = 3/4$. What is the speed of the bullet measured by Earth observers?

(3) Why stop with bullets that saunter along at less than the speed of light? Let the bullet itself be a flash of light. Then the bullet speed as measured in the bus is $v'_{\text{bullet}} = 1$. For $v_{\text{rel}} = 3/4$, with what speed does this light flash move as measured in the Earth frame? Is this what you expect from the Principle of Relativity?

(4) Suppose a light flash is launched from the front of the bus directed toward the back ($v'_{\text{bullet}} = -1$). What is the velocity of this light flash measured in the Earth frame? Is this what you expect from the Principle of Relativity?

Reference: N. David Mermin, *American Journal of Physics,* Volume 51, pages 1130–1131 (1983).

3-12 Michelson–Morley experiment

a An airplane moves with air speed c (not the speed of light) from point A to point B on Earth. A stiff wind of speed v is blowing from B toward A. (In this exercise only, the symbol v stands for velocity in conventional units, for example meters/second.) Show that the time for a round trip from A to B and back to A under these circumstances is greater by a factor $1/(1 - v^2/c^2)$ than the corresponding round trip time in still air. Paradox: The wind helps on one leg of the flight as well as hinders on the other. Why, therefore, is the round-trip time not the same in the presence of wind as in still air? Give a simple physical reason for this difference. What happens when the wind speed is nearly equal to the speed of the airplane?

b The same airplane now makes a round trip between A and C. The distance between A and C is the same as the distance from A to B, but the line from A to C is perpendicular to the line from A to B, so that in moving between A and C the plane flies across the wind. Show that the round-trip time between A and C under these circumstances is greater by a factor $1/(1 - v^2/c^2)^{1/2}$ than the corresponding round-trip time in still air.

c Two airplanes with the same air speed c start from A at the same time. One travels from A to B and back to A, flying first against and then with the wind (wind speed v). The other travels from A to C and back to A, flying across the wind. Which one will arrive home first, and what will be the difference in their arrival times? Using the first two terms of the binomial theorem,

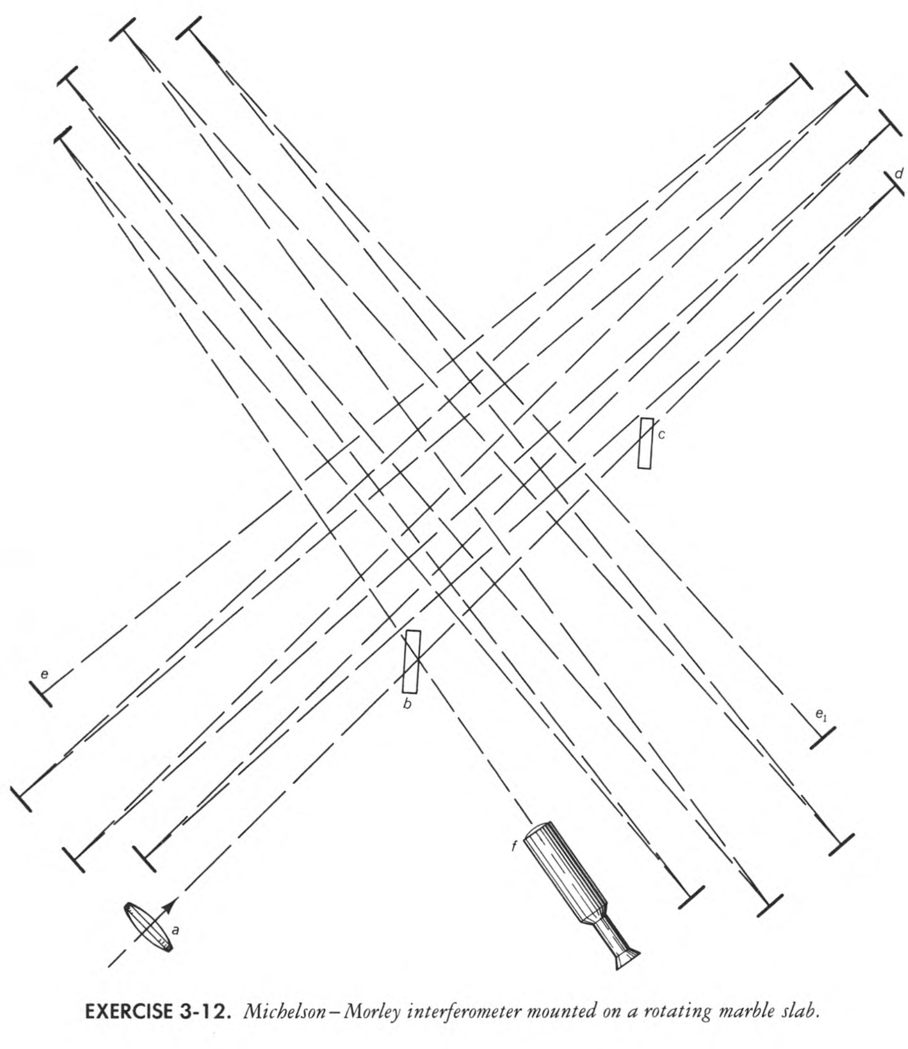
$$(1 + z)^n \approx 1 + nz \qquad \text{for } |z| << 1$$

show that if $v << c$, then an approximate expression for this time difference is $\Delta t \approx (L/2c)(v/c)^2$, where L is the round-trip distance between A and B (and between A and C).

d The South Pole Air Station is the supply depot for research huts on a circle of 300-kilometer radius centered on the air station. Every Monday many supply planes start simultaneously from the station and fly radially in all directions at the same altitude. Each plane drops supplies and mail to one of the research huts and flies directly home. A Fussbudget with a stopwatch stands on the hill overlooking the air station. She notices that the planes do not all return at the same time. This discrepancy perplexes her because she knows from careful measurement that (1) the distance from the air station to every research hut is the same, (2) every plane flies with the same air speed as every other plane—300 kilometers/hour—and (3) every plane travels in a straight line over the ground from station to hut and back. The Fussbudget finally decides that the discrepancy is due to the wind at the high altitude at which the planes fly. With her stopwatch she measures the time from the return of the first plane to the return of the last plane to be 4 seconds. What is the wind speed at the altitude where the planes fly? What can the Fussbudget say about the direction of this wind?

e In their famous experiment Michelson and Morley attempted to detect the so-called **ether drift**—the motion of Earth through the "ether," with respect to which light was supposed to have the velocity c. They compared the round-trip times for light to travel equal distances parallel and perpendicular to the direction of motion of Earth around Sun. They reflected the light back and forth between nearly parallel mirrors. (This would correspond to part **c** if each airplane made repeated round trips.) By this means they were able to use a total round-trip length of 22 meters for each path. If the "ether" is at rest with respect to Sun, and if Earth moves at 30×10^3 meters/second in its path around Sun, what is the approximate difference in time of return between light flashes that are emitted simultaneously and travel along the two perpendicular paths? Even with the instruments of today, the difference predicted by the ether-drift hypothesis would be too small to measure directly, and the following method was used instead.

f The original Michelson – Morley interferometer is diagrammed in the figure. Nearly monochromatic light (light of a single frequency) enters through the lens at *a*. Some of the light is reflected by the half-silvered mirror at *b* and the rest of the light continues toward *d*. Both beams are reflected back and forth until they reach mirrors *e* and e_1 respectively, where each beam is reflected back on itself and retraces its path to mirror *b*. At mirror *b* parts of each beam combine to enter telescope *f* together. The transparent piece of glass at *c*, of the same dimensions as the half-silvered mirror *b*, is inserted so that both beams pass the same number of times (three times) through this thickness of glass on their way to telescope *f*. Suppose that the perpendicular path lengths are exactly equal and the instrument is at rest with respect to the ether. Then monochromatic light from the two paths that leave mirror *b* in some relative phase will return to mirror *b* in the same phase. Under these circumstances the waves entering telescope *f* will add crest to crest and the image in this telescope will be bright. On the other hand, if one of the beams has been delayed a time corresponding to one half period of the light, then it will arrive at mirror *b* one half period later and the waves entering the telescope will cancel (crest to trough), so the image in the telescope will be dark. If one beam is retarded a time corresponding to one whole period, the telescope image will be bright, and so forth. What time corresponds to



EXERCISE 3-12. *Michelson – Morley interferometer mounted on a rotating marble slab.*

one period of the light? Michelson and Morley used sodium light of wavelength 589 nanometers (one nanometer is equal to 10^{-9} meter). Use the equations $f\lambda = c$ and $f = 1/T$ that relate frequency f, period T, wavelength λ, and speed c of an electromagnetic wave. Show that one period of sodium light corresponds to about 2×10^{-15} seconds.

Now there is no way to "turn off" the alleged ether drift, adjust the apparatus, and then turn the alleged ether drift on again. Instead of this, Michelson and Morley floated their interferometer in a pool of mercury and rotated it slowly about its center like a phonograph record while observing the image in the telescope (see the figure). In this way if light is delayed on either path when the instrument is oriented in a certain direction, light on the other path will be delayed by the same amount of time when the instrument has rotated 90 degrees. Hence the total change in delay time between the two paths observed as the interferometer rotates should be twice the difference calculated using the expression derived in part **c**. By refinements of this method Michelson and Morley were able to show that the time change between the two paths as the instrument rotated corresponded to less than one one-hundredth of the shift from one dark image in the telescope to the next dark image. Show that this result implies that the motion of the ether at the surface of Earth — if it exists at all — is less than one sixth of the speed of Earth in its orbit. In order to eliminate the possibility that the ether was flowing past Sun at the same rate as Earth was moving its orbit, they repeated the experiment at intervals of three months, always with negative results.

g Discussion question: Does the Michelson–Morley experiment, by itself, disprove the theory that light is propagated through an ether? Can the ether theory be modified to agree with the results of this experiment? How? What further experiment can be used to test the modified theory?

Reference: A. A. Michelson and E. W. Morley, *American Journal of Science,* Volume 134, pages 333–345 (1887).

3-13 the Kennedy–Thorndike experiment

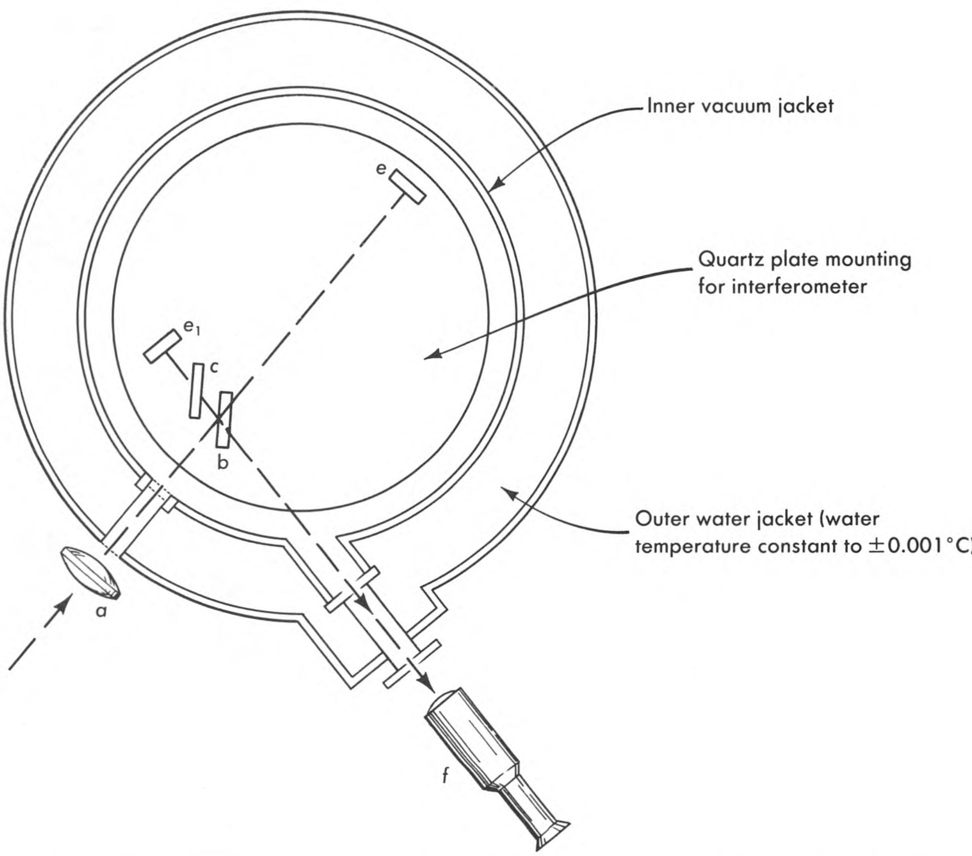
Note: Part **d** of this exercise uses elementary calculus.

The Michelson–Morley experiment was designed to detect any motion of Earth relative to a hypothetical fluid — the ether — a medium in which light was supposed to move with characteristic speed c. No such relative motion of earth and ether was detected. Partly as a result of this experiment the concept of ether has since been discarded. In the modern view, light requires no medium for its transmission. What significance does the negative result of the Michelson–Morley experiment have for us who do not believe in the ether theory of light propagation? Simply this: (1) The round-trip speed of light measured on earth is the same in every direction — the speed of light is isotropic. (2) The speed of light is isotropic not only when Earth moves in one direction around Sun in, say, January (call Earth with this motion the "laboratory frame"), but also when Earth moves in the opposite direction around Sun six months later, in July (call Earth with this motion the "rocket frame"). (3) The generalization of this result to any pair of inertial frames in relative motion is contained in the statement, The round-trip speed of light is isotropic both in the laboratory frame and in the rocket frame. This result leaves an important question unanswered: Does the round-trip speed of light — which is isotropic in both laboratory and rocket frames — also have the same numerical value in laboratory and rocket frames? The assumption that this speed has the same numerical value in both frames played a central role in demonstrating the invariance of the interval (Section 3.7). But is this assumption valid?

a An experiment to test the assumption of the equality of the round-trip speed of light in two inertial frames in relative motion was conducted in 1932 by Roy J. Kennedy and Edward M. Thorndike. The experiment uses an interferometer with arms of unequal length (see the figure). Assume that one arm of the interferometer is Δl longer than the other arm. Show that a flash of light entering the apparatus will take a time $2\Delta l/c$ longer to complete the round trip along the longer arm than along the shorter arm. The difference in length Δl used by Kennedy and Thorndike was approximately 16 centimeters. What is the approximate difference in time for the round trip of a light flash along the alternative paths?

b Instead of a pulse of light, Kennedy and Thorndike used continuous monochromatic light of period $T = 1.820 \times 10^{-15}$ seconds ($\lambda = 546.1$ nanometers $= 546.1 \times 10^{-9}$ meters) from a mercury source. Light that traverses the longer arm of the interferometer will return approximately how many periods n later than light that traverses the shorter arm? If in the actual experiment the number of periods is an integer, the reunited light from the two arms will add (crest-to-crest) and the field of view seen through the telescope will be bright. In contrast, if in the actual experiment the number of periods is a half-integer, the reunited light from the two arms will cancel (crest-to-trough) and the field of view of the telescope will be dark.

c Earth continues on its path around Sun. Six months later Earth has reversed the direction of its velocity relative to the fixed stars. In this new frame of



EXERCISE 3-13. *Schematic diagram of apparatus used for the Kennedy–Thorndike experiment. Parts of the interferometer have been labeled with letters corresponding to those used in describing the Michelson–Morley interferometer (Exercise 3-12). The experimenters went to great lengths to insure the optical and mechanical stability of their apparatus. The interferometer is mounted on a plate of quartz, which changes dimension very little when temperature changes. The interferometer is enclosed in a vacuum jacket so that changes in atmospheric pressure will not alter the effective optical path length of the interferometer arms (slightly different speed of light at different atmospheric pressure). The inner vacuum jacket is surrounded by an outer water jacket in which the water is kept at a temperature that varies less than ±0.001 degrees Celsius. The entire apparatus shown in the figure is enclosed in a small darkroom (not shown) maintained at a temperature constant within a few hundredths of a degree. The small darkroom is in turn enclosed in a larger darkroom whose temperature is constant within a few tenths of a degree. The overall size of the apparatus can be judged from the fact that the difference in length of the two arms of the interferometer (length* eb *compared with length* e_1b*) is 16 centimeters.*

reference will the round-trip speed of light have the same numerical value c as in the original frame of reference? One can rewrite the answer to part **b** for the original frame of reference in the form

$$c = (2/n)(\Delta l/T)$$

where Δl is the difference in length between the two interferometer arms, T is the time for one period of the atomic light source, and n is the number of periods that elapse between the return of the light on the shorter path and the return of the light on the longer path. Suppose that as Earth orbits Sun no shift is observed in the telescope field of view from, say, light toward dark. This means that n is observed to be constant. What would this hypothetical result tell about the numerical value c of the speed of light? Point out the standards of distance and time used in determining this result, as they appear in the equation. Quartz has the greatest stability of dimension of any known material. Atomic time standards have proved to be the most dependable earth-bound time-keeping mechanisms.

d In order to carry out the experiment outlined in the preceding paragraphs, Kennedy and Thorndike would have had to keep their interferometer operating perfectly for half a year while continuously observing the field of view through the telescope. Uninterrupted operation for so long a time was not feasible. The actual durations of their observations varied from eight days to a month. There were several such periods of observation at three-month time separations. From the data obtained in these periods, Kennedy and Thorndike were able to estimate that over a single six-month observation the number of periods n of relative delay would vary by less than the fraction 3/1000 of one period. Take the differential of the equation in part **c** to find the largest fractional change dc/c of the round-trip speed of light between the two frames consistent with this estimated change in n (frame 1 — the "laboratory" frame — and frame 2 — the "rocket" frame — being in the present analysis Earth itself at two different times of year, with a relative velocity twice the speed of Earth in its orbit: 2×30 kilometers/second).

Historical note: At the time of the Michelson–Morley experiment in 1887, no one was ready for the idea that physics — including the speed of light — is the same in every inertial frame of reference. According to today's standard Einstein interpretation it seems obvious that both the Michelson–Morley and the Kennedy–Thorndike experiments should give null results. However, when Kennedy and Thorndike made their measurements in 1932, two alternatives to the Einstein theory were open to consideration (designated here as theory A and theory B). Both A and B assumed the old idea of an absolute space, or "ether," in which light has the speed c. Both A and B explained the zero fringe shift in the Michelson–Morley experiment by saying that all matter that moves at a velocity v (expressed as a fraction of light-speed) relative to "absolute space" undergoes a shrinkage of its space dimensions in the direction of motion to a new length equal to $(1 - v^2)^{1/2}$ times the old length ("Lorentz-FitzGerald contraction hypothesis"). The two theories differed as to the effect of "motion through absolute space" on the running rate of a clock. Theory A said, No effect. Theory B said that a standard seconds clock moving through absolute space at velocity v has a time between ticks of $(1 - v^2)^{1/2}$ seconds. In theory B the ratio $\Delta l/T$ in the equation in part **b** will not be affected by the velocity of the clock, and the Kennedy–Thorndike experiment will give a null result, as observed ("complicated explanation for simple effect"). In theory A the ratio $\Delta l/T$ in the equation will be multiplied by the factor $(1 - v_1{}^2)^{1/2}$ at a time of year when the "velocity of Earth relative to absolute space" is v_1 and multiplied by $(1 - v_2{}^2)^{1/2}$ at a time of year when this velocity is v_2. Thus the fringes should shift from one time of year ($v_1 = v_{\mathrm{orbital}} + v_{\mathrm{Sun}}$) to another time of year ($v_2 = v_{\mathrm{orbital}} - v_{\mathrm{Sun}}$) unless by accident Sun happened to have "zero velocity relative to absolute space" — an accident judged so unlikely as not to provide an acceptable explanation of the observed null effect. Thus the Kennedy–Thorndike experiment ruled out theory A (length contraction alone) but allowed theory B (length contraction plus time contraction) — and also allowed the much simpler Einstein theory of equivalence of all inertial reference frames.

The "sensitivity" of the Kennedy–Thorndike experiment depends on the theory under consideration. In the context of theory A the observations set an upper limit of about 15 kilometers/second to the "speed of Sun through absolute space" (sensitivity reported in the Kennedy–Thorndike paper). In the context of Einstein's theory the observations say that the round-trip speed of light has the same numerical magnitude — within an error of about 3 meters/second — in inertial frames of reference having a relative velocity of 60 kilometers/second.

Reference: R. J. Kennedy and E. M. Thorndike, *Physical Review*, Volume 42, pages 400–418 (1932).

3-14 things that move faster than light

Can "things" or "messages" move faster than light? Does relativity really say "No" to this possibility? Explore these questions further using the following examples.

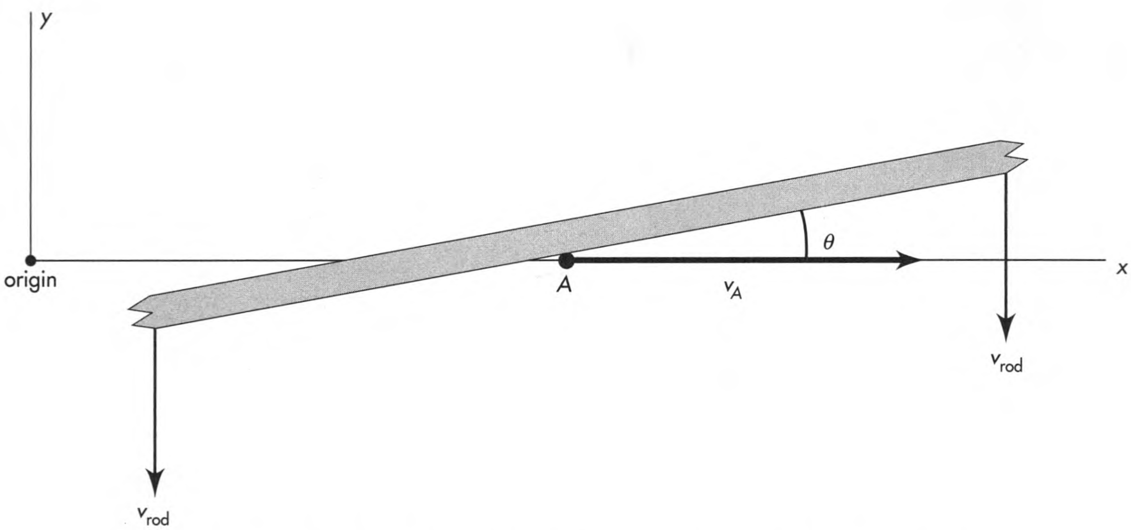
a The Scissors Paradox. A very long straight rod, inclined at an angle θ to the x-axis, moves downward with uniform speed v_{rod} as shown in the figure. Find the speed v_A of the point of intersection A of the lower edge of the stick with the x-axis. Can this speed be greater than the speed of light? If so, for what values of the angle θ and v_{rod} does this occur? Can the motion of intersection point A be used to transmit a message faster than light from someone at the origin to someone far out on the x-axis?

b Transmission of a Hammer Pulse. Suppose the same rod is initially at rest in the laboratory with the point of intersection initially at the origin. The region of the rod centered at the origin is struck sharply with the downward blow of a hammer. The point of intersection moves to the right. Can this motion of the point of intersection be used to transmit a message faster than the speed of light?

c Searchlight Messenger? A very powerful searchlight is rotated rapidly in such a way that its beam sweeps out a flat plane. Observers A and B are at rest on the plane and each the same distance from the searchlight but not near each other. How far from the searchlight must A and B be in order that the searchlight beam will sweep from A to B faster than a light signal could travel from A to B? Before they took their positions, the two observers were given the following instruction:

To A: "When you see the searchlight beam, fire a bullet at B."

To B: "When you see the searchlight beam, duck because A has fired a bullet at you."



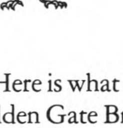
EXERCISE 3-14. *Can the point of intersection* A *move with a speed* v_A *greater than the speed of light?*

Under these circumstances, has a warning message traveled from A to B with a speed faster than that of light?

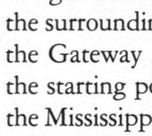
d Oscilloscope Writing Speed. The manufacturer of an oscilloscope claims a writing speed (the speed with which the bright spot moves across the screen) in excess of the speed of light. Is this possible?

3-15 four times the speed of light?

We look westward across the United States and see the rocket approaching us at four times the speed of light.



How can this be, since nothing moves faster than light?



We did not say the rocket *moves* faster than light; we said only that we *see* it moving faster than light.

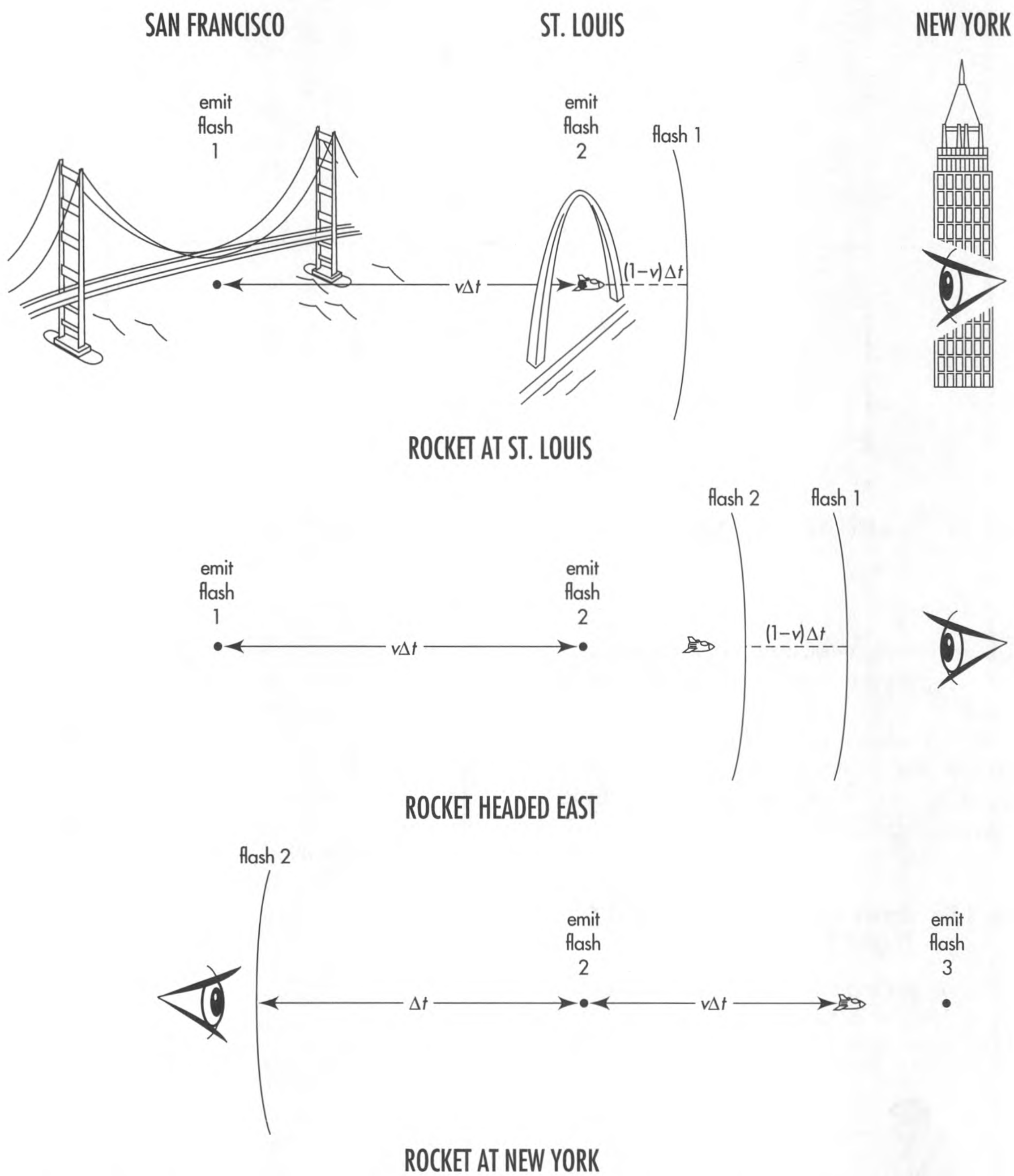
Here is what happens: The rocket streaks under the Golden Gate Bridge in San Francisco, emitting a flash of light that illuminates the rocket, the bridge, and the surroundings. At time Δt later the rocket threads the Gateway Arch in St. Louis that commemorates the starting point for covered wagons. The arch and the Mississippi riverfront are flooded by a second flash of light. The top figure is a visual summary of measurements from our continent-spanning latticework of clocks taken at this moment.

Now the rocket continues toward us as we stand in New York City. The center figure summarizes data taken as the first flash is about to enter our eye. Flash 1 shows us the rocket passing under the Golden Gate Bridge. An instant later flash 2 shows us the rocket passing through the Gateway Arch.

a Answer the following questions using symbols from the first two figures. The images carried by the two flashes show the rocket how far apart in space? What is the time lapse between our reception of these two images? Therefore, what is the apparent speed of the approaching rocket we see? For what speed v of the rocket does the apparent speed of approach equal four times the speed of light? For what rocket speed do we see the approaching rocket to be moving at 99 times the speed of light?

b Our friend in San Francisco is deeply disappointed. Looking eastward, she sees the retreating rocket traveling at less than half the speed of light (bottom figure). She wails, "Which one of us is wrong?" "Neither one." we reply. "No matter how high the speed v of the rocket, you will never see it moving directly away from you at a speed greater than half the speed of light."

Use the bottom figure to derive an expression for the apparent speed of recession of the rocket. When we in New York see the rocket approaching at four times the speed of light, with what speed does our San Francisco friend see it moving away from her? When we see a faster rocket approaching at 99 times the speed of light, what speed of recession does she behold?

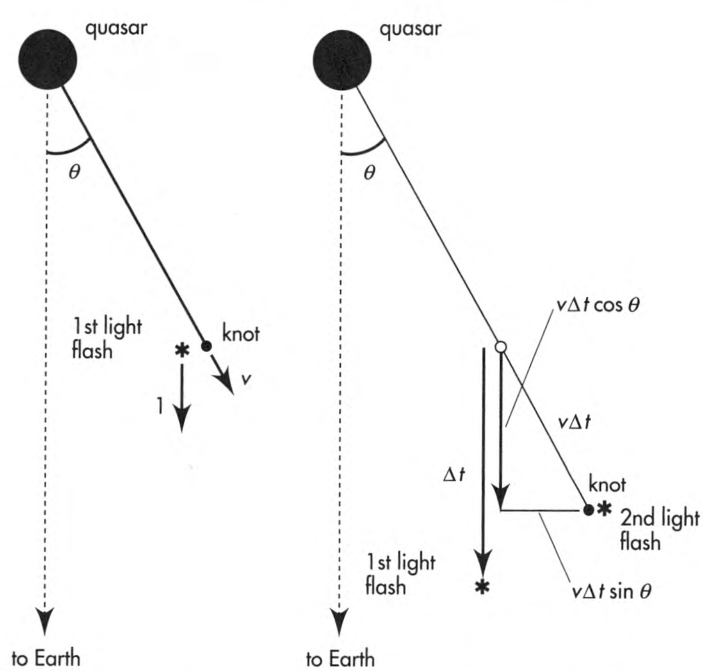


EXERCISE 3-15. ***Top:*** *Rocket headed east, shown at the instant it passes under the Gateway Arch in St. Louis and emits flash 2. The rocket is chasing flash 1, emitted earlier as it passed under the Golden Gate Bridge in San Francisco.* ***Center:*** *The two image-carrying flashes are close together, so they enter the eye in rapid succession. This gives the viewer the visual impression that the rocket moved from San Francisco to St. Louis in a very short time.* ***Bottom:*** *Rocket headed east, shown at the instant it approaches the Empire State Building in New York City and emits flash 3. When the rocket moves away from the viewer, the distance of rocket travel is added to the separation between flashes. This increases the apparent time between flashes, giving the viewer the impression that the rocket moved from St. Louis to New York at less than one half light-speed.*

3-16 superluminal expansion of quasar 3C273?

The most powerful sources of energy we know or conceive or see in all the universe are so-called quasi-stellar objects, or **quasars,** starlike sources of light located billions of light-years away. Despite being far smaller than any galaxy, the typical quasar manages to put out more than 100 times as much energy as our own Milky Way, with its hundred billion stars. Quasars, unsurpassed in brilliance and remoteness, we count today as lighthouses of the heavens.

One of the major problems associated with quasars is that some are composed of two or more components



EXERCISE 3-16, first figure. *Left: Bright "knot" of plasma ejected from a quasar at high speed* v *emits a first flash of light toward Earth.* ***Right:*** *The knot emits a second light flash toward Earth a time* Δt *later. This time* Δt *is measured locally near the knot using the Earth-linked latticework of rods and clocks (har! har!).*

that appear to be separating from each other with relative velocity greater than the speed of light ("superluminal" velocity). One theory that helps explain this effect pictures the quasar as a core that ejects a jet of plasma at relativistic speed. Disturbances or instabilities in such a jet appear as discrete "knots" of plasma. The motion and light emission from a knot may account for its apparent greater-than-light speed, as shown using the first figure.

a The first figure shows two Earth-directed light flashes emitted from the streaking knot. The time between emissions is Δt as measured locally near the knot using the Earth-linked latticework of rods and clocks. Of course the clock readings on this portion of the Earth-linked latticework are not available to us on Earth; therefore we cannot measure Δt directly. Rather, we see the time separation between the arrivals of the two flashes at Earth. From the figure, show that this Earth-seen time separation Δt_{seen} is given by the expression

$$\Delta t_{\text{seen}} = \Delta t(1 - v \cos \theta)$$

b We have another disability in viewing the knot from Earth. We do not see the motion of the knot toward us, only the apparent motion of the knot across our field of view. Find an expression for this transverse motion (call it Δx_{seen}) between emissions of the two light flashes in terms of Δt.

c Now calculate the speed v^x_{seen} of the rightward motion of the knot as seen on Earth. Show that the result is

$$v^x_{\text{seen}} = \frac{\Delta x_{\text{seen}}}{\Delta t_{\text{seen}}} = \frac{v \sin \theta}{1 - v \cos \theta}$$

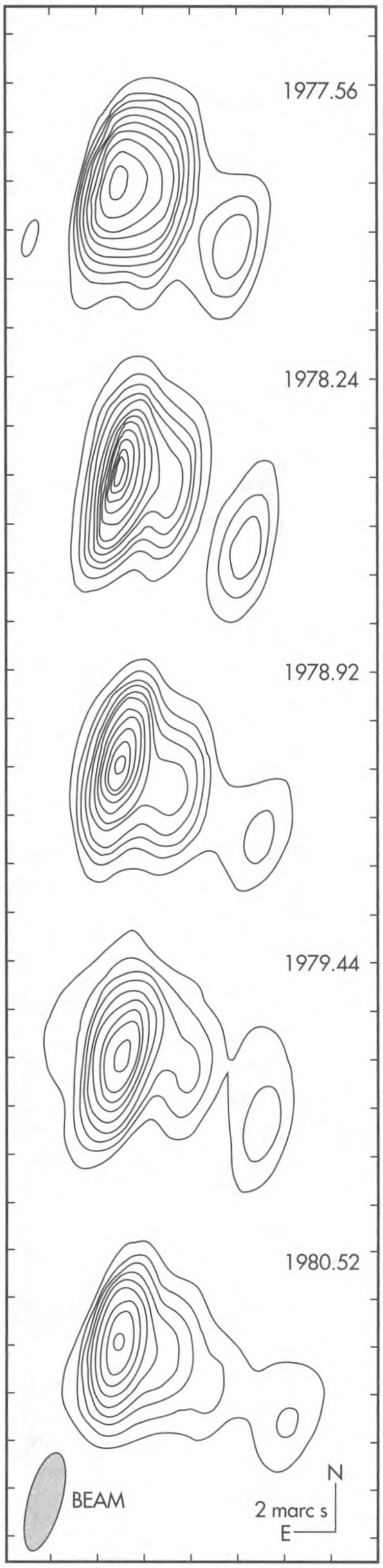
d What is the value of v^x_{seen} when the knot is emitted in the direction exactly toward Earth? when it is emitted perpendicular to this direction? Find an expression that gives the range of angles θ for which v^x_{seen} is greater than the speed of light. For $\theta = 45$ degrees, what is the range of knot speeds v such that v^x_{seen} is greater than the speed of light?

e If you know calculus, find an expression for the angle θ_{max} at which v^x_{seen} has its maximum value for a given knot speed v. Show that this angle satisfies the equation $\cos \theta_{\text{max}} = v$. Whether or not you derive this result, use it to show that the maximum apparent transverse speed is seen as

$$v^x_{\text{seen, max}} = \frac{v}{(1 - v^2)^{1/2}}$$

f What is this maximum transverse speed seen on Earth when $v = 0.99$?

g The second figure shows the pattern of radio emission from the quasar 3C273. The decreased pe-



EXERCISE 3-16, second figure. *Contour lines of radio emission from the quasar 3C273 showing a bright "knot" of plasma apparently moving away from it at a speed greater than the speed of light. The time of each image is given as calendar year and decimal fraction. Horizontal scale divisions are in units of 2 milli arc-seconds. (1 milli arc-second = $10^{-3}/3600$ degree = 4.85×10^{-9} radian)*

riod of radiation from this source (Exercise 3-10) shows that it is approximately 2.6×10^9 light-years from Earth. A secondary source is apparently moving away from the central quasar. Take your own measurements on the figure. Combine this with data from the figure caption to show that the apparent speed of separation is greater than 9 times the speed of light.

Note: As of 1990, apparent greater-than-light-speed ("superluminal") motion has been observed in approximately 25 different sources.

References: Analysis and first figure adapted from Denise C. Gabuzda, *American Journal of Physics,* Volume 55, pages 214–215 (1987). Second figure and data taken from T. J. Pearson, S. C. Unwin, M. H. Cohen, R. P. Linfield, A. C. S. Readhead, G. A. Seielstad, R. S. Simon, and R. C. Walker, *Nature,* Volume 290, pages 365–368 (2 April 1981).

3-17 contraction or rotation?

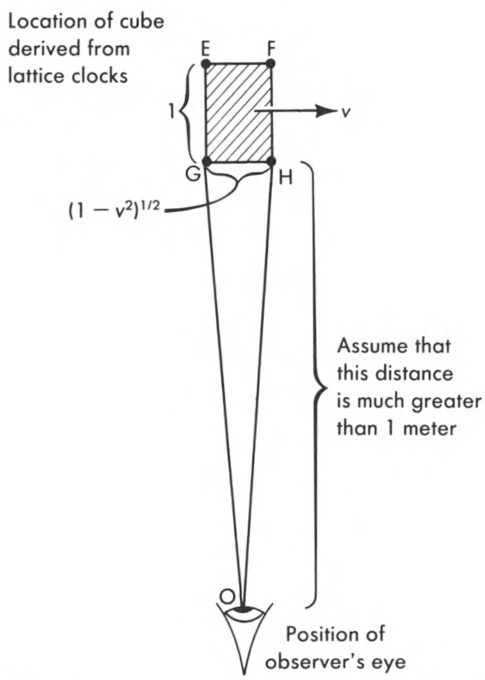
A cube at rest in the rocket frame has an edge of length 1 meter in that frame. In the laboratory frame the cube is Lorentz contracted in the direction of motion, as shown in the figure. Determine this Lorentz contraction, for example, from locations of four clocks at rest and synchronized in the laboratory lattice with which the four corners of the cube, E, F, G, H, coincide when all four clocks read the same time. This latticework measurement eliminates time lags in the travel of light from different corners of the cube.

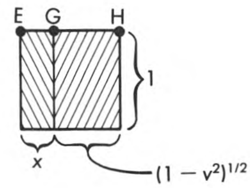
Now for a different observing procedure! Stand in the laboratory frame and look at the cube with one eye as the cube passes overhead. What one sees at any time is light that enters the eye at that time, even if it left the different corners of the cube at different times. Hence, what one sees visually may not be the same as what one observes using a latticework of clocks. If the cube is viewed from the bottom then the distance GO is equal to the distance HO, so light that leaves G and H simultaneously will arrive at O simultaneously. Hence, when one sees the cube to be overhead one will see the Lorentz contraction of the bottom edge.

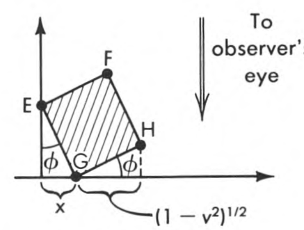
a Light from E that arrives at O simultaneously with light from G will have to leave E earlier than light from G left G. How much earlier? How far has the cube moved in this time? What is the value of the distance x in the right top figure?

b Suppose the eye interprets the projection in the figures as a rotation of a cube that is not Lorentz contracted. Find an expression for the angle of apparent rotation ϕ of this uncontracted cube. Interpret this expression for the two limiting cases of cube speed in the laboratory frame: $v \rightarrow 0$ and $v \rightarrow 1$.

c **Discussion question:** Is the word "really" an appropriate word in the following quotations?







EXERCISE 3-17. ***Left:*** *Position of eye of visual observer watching cube pass overhead.* ***Right top:*** *What the visual observer sees as she looks up from below.* ***Right bottom:*** *How the visual observer can interpret the projection of the second figure.*

(1) An observer using the rocket latticework of clocks says, "The stationary cube is really neither rotated nor contracted."

(2) Someone riding in the rocket who looks at the stationary cube agrees, "The cube is really neither rotated nor contracted."

(3) An observer using the laboratory latticework of clocks says, "The passing cube is really Lorentz contracted but not rotated."

(4) Someone standing in the laboratory frame looking at the passing cube says, "The cube is really rotated but not Lorentz contracted."

What can one rightfully say — in a sentence or two — to make each observer think it reasonable that the other observers should come to different conclusions?

d The analysis of parts **b** and **c** assumes that the visual observer looks with one eye and has no depth perception. How will the cube passing overhead be perceived by the viewer with accurate depth perception?

Reference: For a more complete treatment of this topic, see Edwin F. Taylor, *Introductory Mechanics* (John Wiley and Sons, New York, 1963), pages 346–360.