

THE WAVEFUNCTION

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1. INTRODUCTION

Now we use the Sum Over Paths theory of quantum mechanics to introduce the major player in conventional quantum mechanics: the **wavefunction** for the electron.

What is the wavefunction? Up until now we have considered a single detector that may (or may not) CLICK, signaling detection of the electron at a given time. Our computation of a resulting arrow for that detection event helps us to predict (from its squared magnitude) the probability of detecting the electron there and then.

But why be satisfied with a single detector and a single detection event? Why look at one little piece of space—one detector—when we can raise our eyes and see the entire vicinity? Why not have detectors all over the place and ask what the probability is of detecting the electron ANYWHERE IN SPACE AT A GIVEN TIME?

No problem! We just have to recalculate for that time (but for different locations) the resulting arrow at each of the detectors spread throughout space. Different paths to different locations usually mean different resulting arrows. The result is a whole collection of resulting arrows, the squared magnitude of each telling us the relative probability of finding the electron at that detector at that time.

The wavefunction at a given time t is the whole collection of resulting quantum arrows spread over space at that time.

Where IS the particle at time t ? Nobody knows! It has not been detected yet. But the wavefunction tells us where to look: The relative probability of finding the particle at each location at time t is proportional to the squared magnitude of the resulting arrows at that location.

2. THE SPACE WINDOW

(The following is a BIT complicated. Hang in there -- it is important!)

Where does the particle start? Up until now we have assumed that the particle is emitted from a single source (a unique point) at a particular time. (The only exception was the photon emitted over time from a coherent source—Figure 68, page 104 of QED.) Now we want to generalize the idea of initial conditions. Think of the following sequence of events:

STEP 1. An electron is emitted from a source at an earlier time t_{source} .

STEP 2. At the present moment, t_{now} , we use a bright light to look for the electron

everywhere EXCEPT in a reserved region of space. Call the **Space Window** the reserved region of space at time t_{now} , where we do NOT look for the electron.

STEP 3. IF we detect the electron in one of the places where we DO look at t_{now} , then we discontinue the experiment and start again with Step 1. IF we do NOT find the electron, we assume that the electron paths pass through the Space Window. Then continue the present thought experiment with Step 4.

In Step 3 we find out something—a range of positions where the electron must be—by NOT looking for it there. Crazy world, right?

Figure 1 summarizes the situation so far for one space dimension.

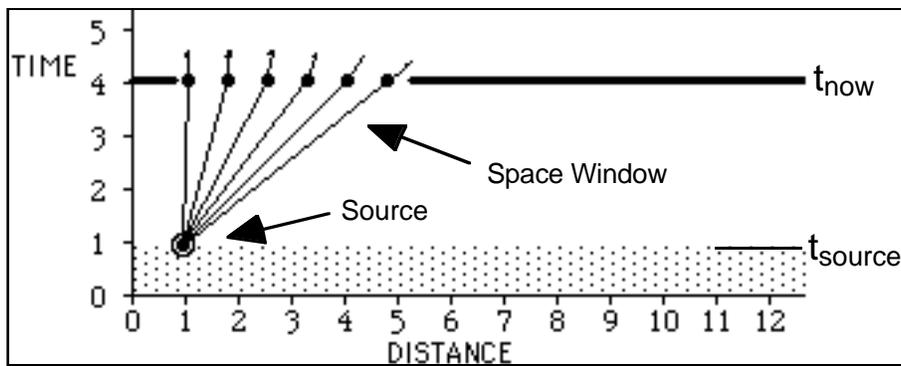


Figure 1: The Space Window in one space dimension. The Space Window is the opening in the heavy black horizontal line drawn at time t_{now} . The heavy black line represents the fact that we have not found the electron by looking everywhere with a bright light EXCEPT at the range of locations occupied by the Space Window. Worldlines connect the source with points in the Space Window. The little arrows on dots in the Space Window represent the quantum clock readings at those points in space and time.

STEP 4. We want to predict possible locations of the electron at a particular time t_{future} in the future. To do so, we construct all possible worldlines from the initial emission event to every point in space at time t_{future} . With our bright light, we eliminated all worldlines that do not pass through the Space Window. So we assume that all possible worldlines between t_{source} and t_{future} pass through the Space Window (the unexamined region of space at time t_{now}). See Figure 2.

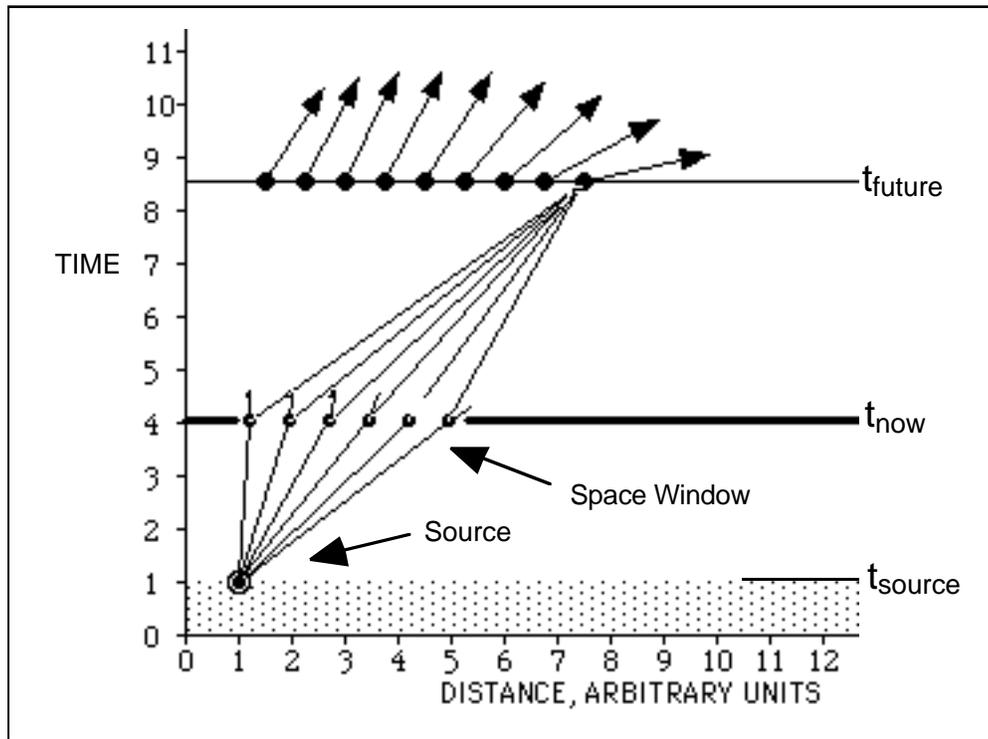


Figure 2: To reach the future, the electron explores only those worldlines that pass through the Space Window.

3. PREDICTING THE FUTURE STARTING NOW

Continue the thought experiment described in the preceding section: In the Space Window, construct the wavefunction at t_{now} . Every point in the Space Window has a quantum clock reading—a little arrow pointing in some direction—resulting from the connection between this event and the single emission event at the earlier time t_{source} .

Our final wavefunction at the later time t_{future} was ENTIRELY constructed using ONLY worldlines that passed through the Space Window. No one can stop us from focusing our attention on the last half of this thought experiment, looking only at the worldlines that go from events in the space window at time t_{now} to the final point of detection at t_{future} . Then the last half of the experiment can be thought of as the evolution of a wavefunction that starts in the Space Window at t_{now} . Starting now, the wavefunction develops until it describes arrows everywhere in space at a later time t_{future} . See Figure 3.

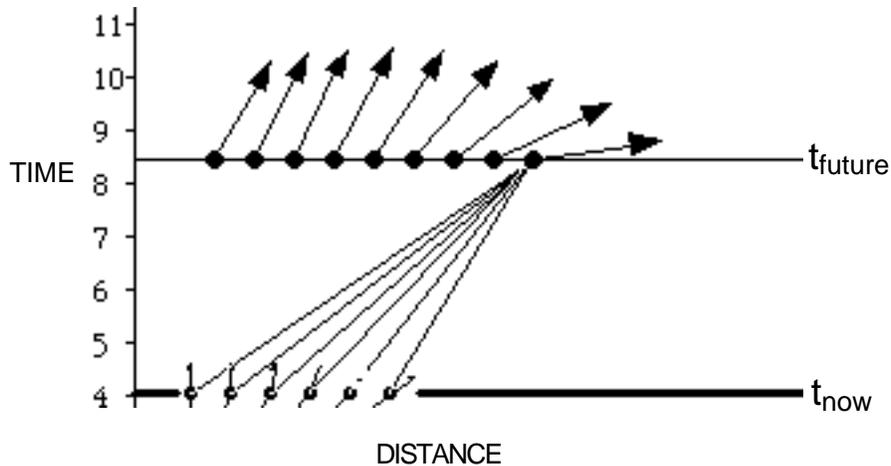


Figure 3: We can construct the wavefunction in the future starting from the wavefunction in the Space Window. Worldlines originating from the source are no longer needed.

REMINDER: These wavefunctions at different times all represent a SINGLE electron.

4. AN ARBITRARY INITIAL WAVEFUNCTION

The wavefunction at time t_{now} described in the preceding section still represents a specialized situation: an electron emitted from a point source at an earlier time t_{source} . But the earlier emission of the electron from a point may not be accessible to us. An electron inside a hydrogen atom in intergalactic space may have been in that one atom for centuries. So to describe Nature we need to be able to start with an initial wavefunction that represents the situation NOW (at t_{now}) in order to predict the situation in the FUTURE (at t_{future}). And our thought experiment shows us how to do this: Just start with some initial wavefunction (initial arrows assigned to every point in some region of space at t_{now}), as shown in Figure 4. Then use the Sum Over Paths theory to connect every one of these points at time t_{now} to each point in space at some future time t_{future} , similar to what is shown in Figure 3 for our special case. The resulting arrows at time t_{future} tell us all that quantum mechanics can about the probability of finding the electron at different locations at this future time t_{future} .

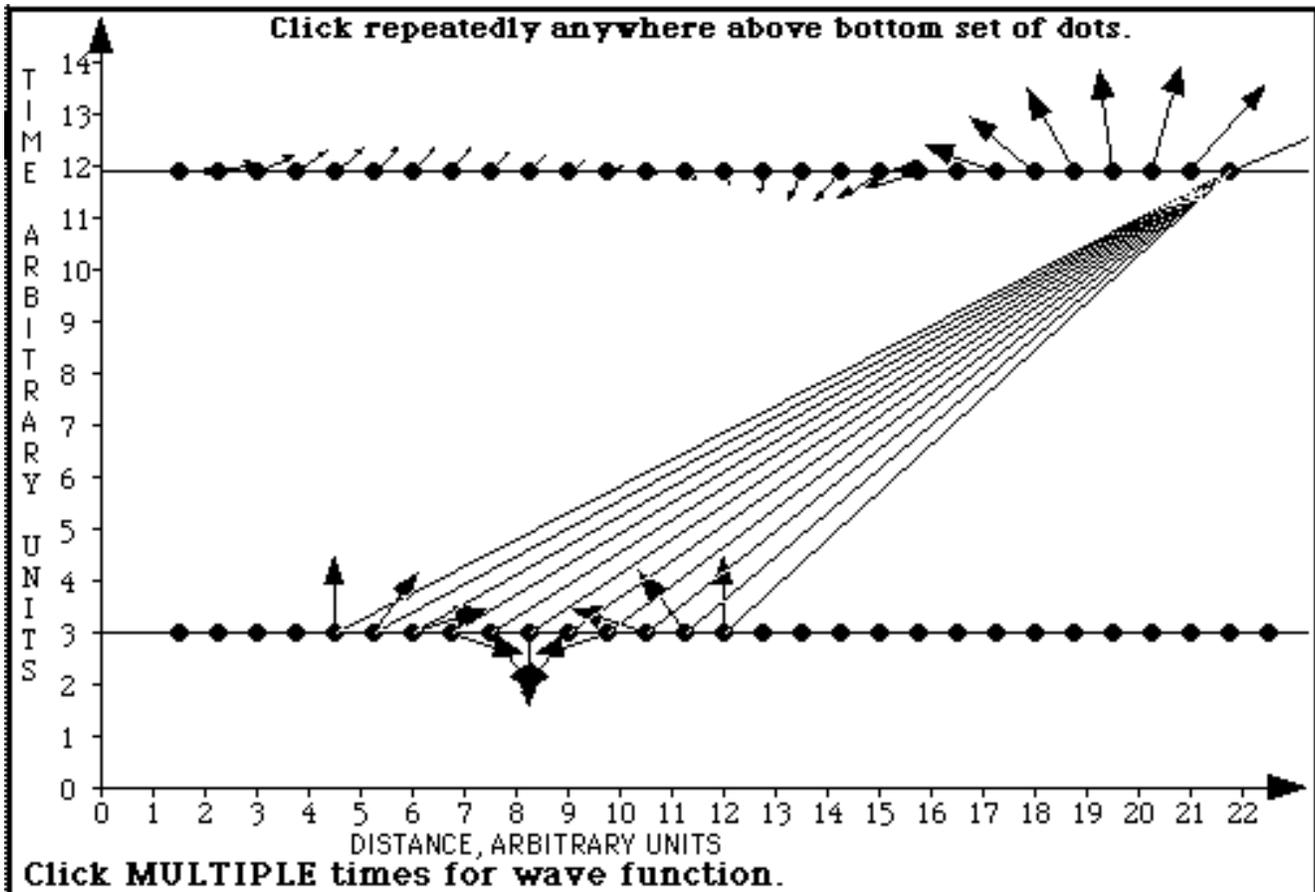


Figure 4. An extended arbitrary initial wavefunction now has a life of its own, with the sum over paths formulation telling it how to propagate forward in time. Here a packet moves to the right.

In brief, the Sum Over Paths theory allows us to start with a wavefunction at time t_{now} (representing a single electron in some region of space) and end with a prediction of the wavefunction at some later time t_{future} (representing the same single electron at the later time).

When we decide to look for the electron at this future time, the squared magnitude of the value of the wavefunction at any point tells us the probability of finding the electron at that point.

That is the essence of nonrelativistic quantum mechanics.

5. THE WAVEFUNCTION SOFTWARE

The homework uses the WAVEFUNCTION program. Here are some introductory activities to get you used to the software. (Do not turn in anything having to do with Step 0 thru Step 10, though you may discuss them with other class members.)

STEP 0. Start up the WAVEFUNCTION program.

STEP 1. Click the BACKGROUND button and read the text. You may want to return to this after you have played with the program some more.

STEP 2. Click on DONE to move on to the initial screen. This shows a spacetime diagram with the time axis vertical.

STEP 3. Click on the INITIAL EVENT button, then click near the bottom of the spacetime diagram to create the initial emission event.

STEP 4. Now create the Space Window. To do this, click in the yellow region above the initial event and to the left of it. Then click to the right of the initial event. This creates a “window,” a horizontal barrier at that time, with a gap or window thru which all worldlines must pass from the initial event to all positions at later times.

STEP 5. Now click above the space window to show the wavefunction at a later time. This wavefunction is sampled at a row of equally spaced detectors. The arrow at each point is the resulting arrow from paths that connect the initial event with this later event. These paths all go through the Space Window.

STEP 6. Click at different times (different vertical distances above the Space Window) to observe the wavefunction at these different times.

STEP 7. Click on the PROBABILITY button to see the probability function corresponding to the wavefunction. Then click on RUN TIME to see a movie, or click on STEP TIME to see each movie frame in sequence.

STEP 8. Press the NEW CASE button and read the various briefing notes given under the Notes menu at the top of the screen. (These briefing notes are available only when you start a new case.)

STEP 9. Play around with steps 1 thru 8, trying various different setups.

STEP 10. After pressing the NEW CASE button, look at the **What’s Wrong** menu item. This will tell you how this software goes only part way to representing real quantum particles.

7. EXERCISES WITH THE WAVEFUNCTION SOFTWARE

For questions Q1 thru Q5, build various Space Windows, each about 7 units wide as measured along the horizontal axis. (NOTE: In all these displays there will be arrows and humps scattering around all over the place. When we speak of “the pulse” we mean one of the primary pulses.) Conceptual problems? See Section 8. Dialogs with Stephanie and George, which follows this section.

Click on an initial event near $x = 0$, $t = 0$ (near the lower left corner of the spacetime graph). With your next click at $x = 4$, $t = 5$ locate the left-hand side of the space window. Next click

on the right side of the space window at $x=11$, $t = 5$. Now watch the development with time by two methods: METHOD A: Click at a series of later times higher up on the spacetime diagram. METHOD B: Click on PROBABILITY, then RUN TIME and watch the probability develop with time

Q1. Describe BRIEFLY the behavior of the wavefunction arrows as observed by Methods A and B for this particular setup. In which direction does the largest pulse move?

Q2. As the pulse of Q1 moves with time, does the largest probability pulse get narrower, wider, or stay the same width?

Q3. Start a new case and follow the instructions of Q1 again (initial event near $t = 0$ and Space Window at about $t = 5$). This time change the relative horizontal positions of initial event and 7-unit-wide Space Window so that the resulting pulse moves to the right FASTER than the one you constructed in Q1. Describe BRIEFLY how you did this.

Q4. Look at the little of arrows in the Space Window (seven units wide) for the case of a pulse that moves to the right. How does the direction of each arrow in the sequence differ as you move from left to right in the gap?

Q5. Continuing the case of Q4, how do the relative directions between arrows in the gap differ for a pulse that moves faster?

Start a new case and find the already-constructed wavefunctions by clicking on the INITIAL WFCN button on the initial display screen. In each case, make a PREDICTION in advance about how the wavefunction will develop in time. GUESS! Then check your prediction by trying it out.

Q6. For each of the given initial wavefunctions, describe how it develops in time:

A. Are there major and minor pulses?

B. How do each of the major/minor pulses move with time?

C. Does the major/minor pulse spread as it moves? Does it contract?

NEXT try constructing initial wavefunctions for yourself, using the BUILD OWN WF button at the bottom when you initially press the INITIAL WFN button. You can either modify one of the given initial wavefunctions or press the button BUILD WF FROM SCRATCH. Note: In constructing your own wavefunction, you do not need to have all the arrows (or ANY arrows!) vertically up or down—drag them to any angle you want.

Q7. Construct an initial wavefunction that leads to a main pulse moving to the LEFT on the screen. (Suggestion: Repeat questions Q1 through Q3, this time to find a pulse that moves to the left, and look at the relative directions of the arrows in the Space Window. Then start again and construct your initial wavefunction.) BRIEFLY describe your initial wavefunction.

8. DIALOGS WITH STEPHANIE AND GEORGE

(lightly edited and class members' names changed)

Stephanie: A rough week so far, so this will be short. I'm struggling a lot with the reading and assignment so far. The contributions by others has helped, but I can't seem to follow what's happening in the software yet. What does the movement of the probability pulse to the right signify? See? I'm still on #1! Any insights???

Steph

Edwin: You are troubled by a truly fundamental question. So far we have dealt with the probability of detection at a specific single detection event. Experiment may find the particle there or not. But the particle must be SOMEWHERE at that time. So there must be a probability of finding it elsewhere. The squared wave function (squared lengths of each of those little arrows) tells us this. When the probability forms a rounded pulse, this means that it we are more likely to find the particle near the center of the pulse than near the edges. And the motion of probability with time says that as time goes on, the particle is more likely to be found farther along in that space direction.

Does this help any? Edwin

Stephanie: Yes. After posting this yesterday and rethinking, I began seeing it along those lines. It leads me to yet another question: Why does it move to the right over time? Probably another fundamental question.

Anyway, thanks for the help. I'm Sloooooowly getting the idea, I think.

Steph

Edwin: You are right: Another fundamental question! The word "why" is always tricky in science. To be more specific: What is it about the position of the space window and (as a result) the little arrows in the space window that leads to the pulse moving to the right? How could you do something similar to make a pulse move to the left? And what do the little arrows do in the space window in THAT case?

After analyzing the space window cases, can you CONSTRUCT an initial wave function that will lead to a pulse that moves to the left?

Edwin

GEORGE WRITES

George: What is a pulse?

Edwin: It is a set of adjacent quantum arrows at a given time that reach a maximum in some region of space. Their squared magnitudes tell us that the probability of finding the particle also reaches a maximum in that region in space. This is the best that quantum mechanics can do to describe the motion of a free particle.

George: What insight are we supposed to gain by considering the speed of the pulse?

Edwin: How to create a quantum pulse that moves with different speeds, and how differences in the initial wave function (at the space window) lead to these different speeds.

George: What IS the connection between the initial wave function pattern and the probability pulse?

Edwin: One leads to the other by repeated application of the Many Paths theory.

9. RESEARCH PROJECT

FINALLY, develop a brief research project of your own. Pose a question about wavefunctions or the resulting probabilities and use the software to answer that question. Write a BRIEF report on your research question and the results of your investigation. Maximum credit for this research project is equal to the credit for five ordinary questions. Possible topics, to give you ideas:

TOPIC A: Reverse all arrows. Suppose that I reverse the direction of every arrow in an initial wavefunction while keeping its length the same. What difference does this make in the resulting wavefunction at a later time? What difference does it make in the initial probability profile? In the probability profile at a later time? How can I account for these similarities and differences?

TOPIC B: Rotate all arrows. Same as TOPIC A, but with the condition that I change an initial wavefunction by rotating each arrow by the same angle while keeping its length the same.

TOPIC C: Zero at the middle. What initial wavefunction gives a value zero at the center for all later times. What are different kinds of wavefunctions that have this property?

TOPIC D: Rate of spreading. What sort of initial wavefunction produces a probability pulse that spreads most slowly. In particular, does an initial narrow bell-shaped wavefunction stay more focused than a wide bell-shaped curve? Or the other way around?

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