

DEMYSTIFYING QUANTUM MECHANICS

A WORKBOOK

Edwin F. Taylor

**Department of Physics, Emeritus
Massachusetts Institute of Technology**

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And then there is Carla . . .

Quantum Mechanics Workbook

Edwin F. Taylor

This workbook follows a “story line” that introduces central features of Feynman's sum-over-paths approach to quantum mechanics. There are many references to his book, on which the treatment is based.

QED, The Strange Theory of Light and Matter, Richard P. Feynman (with Ralph Leighton), Princeton University Press, 1985. Paperback ISBN 0-691-02417-0.

See also Suggested Readings on a later page.

The following article summarizes for a professional audience the story line of this book: “Teaching Feynman’s sum-over-paths quantum theory,” *Computers in Physics*, Volume 12, Number 2, March/April 1998, pages 190-199.

Much of this workbook consists of computer exercises for students to carry out in learning the sum-over-paths approach to quantum mechanics. Software is an essential part of this treatment. Earlier drafts of these programs and exercises have been used by others in classes at the University of Washington and Carnegie Mellon University. They have also been used multiple times in an internet computer conference course from Montana State University, populated largely by high school science teachers.

The web page for the internet courses is: <http://btc.montana.edu/nten>
Or contact Kelly Boyce at kboyce@montana.edu, phone (800) 282-6062.

NOTE: These are draft materials (as is the accompanying software), NOT in their final form, and currently revised every few months.

The latest versions of exercises and software may be downloaded free of charge from the website <http://www.eftaylor.com/>

Or contact Edwin Taylor

Email: eftaylor@mit.edu

Postal address: PO Box 401, Arlington, MA 02476, USA

CONTENTS

Acknowledgments	(back of title page)
Introduction	(preceding page)
Suggested Readings in Quantum Mechanics	2 and 3
Synoptic Contents of Feynman's QED book	4
Old Ideas—and the New Ideas of QED	5
“Wave vs. Particle,” “Explain,” and “Reality”	6
Exercises for the “Reflect” Software Program	7 thru 12
Photon exercises for the “OneParticle” Software Program	13 thru 22
Quantum Stopwatch for the Free Electron	23 thru 30
The Principle of Least Action	31 thru 42
The Electron in a Potential	43 thru 50
Worldlines for the Quantum Particle	51 thru 58
The Wave Function	59 thru 68
The Propagator for a Free Particle	69 thru 82
The Simple Harmonic Oscillator	83 thru 88
Feynman Diagrams	89 thru 94
Single Photon Interference in 1909 A paper by G. I. Taylor	95
“The Principle of Least Action” by R. P. Feynman (excerpt from Chapter 19, Vol. II of <i>The Feynman Lectures on Physics</i>)	96 thru 101
“Teaching Feynman's Sum Over Paths Quantum Theory” Reprint: <i>Computers in Physics</i> , 12 (2), March/April 1998	Journal pages 190 thru 199
Selected Physical Constants	(inside back cover)

Suggested Readings in Quantum Mechanics

Note: All ISBN numbers for the following books are for the *paperback* editions, when available.

Popular Books

There is a TON of popular books on quantum mechanics. Look over the science bookshelf at your local library or bookstore. If you have a favorite, post the title for the other participants. My current favorites are:

QED, The Strange Theory of Light and Matter, Richard P. Feynman, Princeton, 1985, ISBN 0-691-02417-0. This is our text, and a miniature wonder. Feynman gives the quantum commands in the simple form required if the brainless electron and photon are to understand and obey them. That is the good news. The bad news is that there is apparently no book that starts from this simple treatment and develops it further for the serious beginning student of the subject. Beyond this popular book, all we have is graduate texts.

Schrodinger's Kittens and the Search for Reality: Solving the Quantum Mysteries by John Gribbin, Little Brown, 1995, ISBN 0-316-32838-3. Beautifully written. An excellent overview of quantum mechanics and its modern applications and paradoxes. Excellent also is the presentation of the alternative interpretations of the theory. The author apparently prefers the *transactional theory* of quantum mechanics. If you purchase a single additional book for this course, this is the one.

The Meaning of Quantum Theory, by Jim Baggott, Oxford, 1992, ISBN 0-19-855575-X. The same kind of treatment as that of Gribbin, but with some equations.

Quantum Texts

Six Ideas that Shaped Physics, Unit Q: Matter Behaves like Waves, by Thomas A. Moore, McGraw Hill 1998, ISBN 0-07-043057-8. My favorite very recent introduction to quantum mechanics.

Modern Physics for Scientists and Engineers, by J. R. Taylor and C D. Zafiratos, Prentice-Hall, 1991. Another fairly recent introductory quantum text.

Modern Physics, by Paul A. Tipler, Worth, 1978, ISBN 0-87901-088-6. Beautifully written, but out of date.

An Introduction to Quantum Physics, by A. P. French and Edwin F. Taylor, Norton, 1978, ISBN 0-393-09106-0. Beautifully written, but out of date.

The Physics of Atoms and Quanta, by Hermann Haken and Hans Christoph Wolf, Fourth Edition, Springer-Verlag, 1994, ISBN 0-387-57874-9. Up to date and comprehensive, but too mathematical for this course.

Introduction to Quantum Mechanics by David J. Griffiths, Prentice Hall, 1995, ISBN 0-13-124405-1. Thought by many to be the best current junior level quantum text.

Advanced Texts

Quantum Mechanics and Path Integrals, by R. P. Feynman and A. R. Hibbs, McGraw-Hill, 1965 (no ISBN number). This is Feynman's attempt to start with the sum over paths theory of quantum mechanics and then splice onto the conventional Schroedinger wave theory.

Techniques and Applications of Path Integration, by L. S. Schulman, John Wiley, 1996, ISBN 0-471-16610-3. A more recent treatment of the material in Feynman and Hibbs.

Principles of Quantum Mechanics, 2nd ed., by R. Shankar, New York, Plenum Press, 1994. This includes a nice introduction of the sum over paths theory and many applications, suitable for an upper undergraduate or graduate course.

An Original Article and the Nobel Prize Speech

In 1948, Richard Feynman wrote a review article that laid out the story line we are following in this course. If you have an urge to go back to the roots of a subject, this article is for you.

R. P. Feynman, "Space-time Approach to Non-Relativistic Quantum Mechanics," *Reviews of Modern Physics*, Volume 20, Number 2, April 1948, pages 367 thru 387.

And here is Feynman's Nobel Prize speech, in which he traces the history of his work in the field.

R. P. Feynman, "The Development of the Space-Time View of Quantum Electrodynamics," *Science*, Vol. 153, Number 3737 (12 August 1966), pages 699-708.

Biography

You can pick up a good deal of the subject by watching the originators struggle with the difficulties that led to success and Nobel Prizes.

Richard Feynman, A Life in Science, John Gribbin and Mary Gribbin, Dutton, 1997, ISBN 0-525-94124-X. A new biography, a bit more chatty than Gleick.

Genius, James Gleick, Random House, 1992, ISBN 0-679-74704-4. A beautifully written biography of Richard Feynman, with a good deal of science content but no equations.

The Beat of a Different Drum: The Life and Science of Richard Feynman, by Jagdish Mehra, Oxford, 1994, ISBN 0-19-853948-7. Written by a colleague of Feynman, more serious about the science, with some equations.

QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga, by Silvan Schweber, Princeton, 1994, ISBN 0-691-03327-7. If you want the FULL historical treatment, with the FULL mathematical formalism, this is your book. The rest of us can learn a lot by "reading around the edges" of the mathematics.

Synoptic Contents of

QED: The Strange Theory of Light and Matter

by Richard P. Feynman

(Princeton University Press, 1985)

(Seventh printing, with corrections, 1988)

Courtesy of Daniel F. Styer

Chapter 1: Introduction	
Introduction.....	3-9
Approach taken by these lectures.....	9-12
Light and photons.....	13-16
Description of partial reflection.....	16-24
Calculation of partial reflection through probability amplitudes.....	24-33
Iridescence.....	33-35
Chapter 2: Photons: Particles of Light	
Review and preview.....	36-38
Reflection from mirrors.....	38-45
Diffraction gratings.....	45-49
Other properties of light.....	50-59
Compound events.....	59-63
Partial reflection again.....	64-72
Concomitant events.....	72-76
Chapter 3: Electrons and Their Interactions	
Review.....	77-78
Measurement.....	78-83
The three basic actions.....	83-85
Space-time diagrams.....	86-87
Amplitudes for the three basic actions.....	88-92
Electron-electron scattering, diagrammatic perturbation theory.....	92-97
Electron-photon scattering, antiparticles.....	97-99
Atoms.....	99-100
Partial reflection of photons from a fundamental point of view.....	101-107
Transmission of photons through media from a fundamental point of view.....	107-110
Photon clumping (bosons, stimulated emission).....	110-112
Exclusion principle, chemistry, solid state physics.....	112-114
Magnetic moment of the electron.....	115-119
Review.....	119-120
Polarization.....	120-122
Classical limit.....	122-123
Chapter 4: Loose Ends	
Introduction.....	124-125
Renormalization.....	125-129
Origin of the coupling constants.....	129-130
High energy physics.....	130-152

Table 1: Old Ideas—and the New Ideas of QED

Page numbers refer to Feynman's book *QED, The Strange Theory of Light and Matter*

OLD IDEA	Page	NEW IDEA	Page
Light consists of waves.		Light is energy “hunks” called photons.	15
Photons reflect the from front surface and back surface of a sheet of glass.	16	Photons are scattered by electrons throughout the glass	104
A photon or electron moves along a straight-line path from source to detector.		A photon or electron explores ALL paths between source and detector.	37f
The photon “quantum stopwatch” hand rotates as the photon explores each path.	27	Photon clock hand remains stationary as photon explores path. Initial position of clock hand depends on initial emission time from source. (Electron “quantum stopwatch” hand DOES rotate as electron explores path.)	102
A “path” (explored by photon or electron) means a trajectory in space.		A “path” means a trajectory in space PLUS location of particle on that trajectory at each time. In relativity-talk, a path is a “worldline in the spacetime diagram.”	86
A free electron (no gravity or electrical force) has a constant kinetic energy along the path.		A free electron exploring many paths may have different kinetic energies along different paths and along different portions of a single path.	
An electron moves in an atom under a force provided by the electromagnetic field of the nucleus.		An electron explores all paths in an atom, exchanging virtual photons with the nucleus and with itself (NO field!).	100
Photons do not interact with one another.		Photons tend to “clump” as a result of the statistics that they follow (Bose-Einstein statistics). Feynman calls this a “polarization” effect.	111
Electrons repel one another because of their charge.		Yes, but also electrons avoid one another as a result of the statistics that they follow (Fermi-Dirac statistics). Feynman calls this a “polarization” effect.	112

“Wave vs. Particle,” “Explain,” and “Reality”

People argue a lot about fundamental ideas in quantum mechanics. Dan Styer of Oberlin College has made insightful comments on two such subjects. At the end is an opinion about a third subject by Taylor. PLEASE UNDERSTAND that these comments are personal OPINIONS—cocktail-party talk!

Wave vs. Particle (Styer)

I think that Feynman makes a pedagogical error on page 15 of *QED, The Strange Theory of Light and Matter* when he insists that light is a particle. It is certainly true that you will always get the correct answer by considering light to be a particle . . . a particle that behaves in the strange quantal way. But the word “particle” suggests an object that behaves in the familiar baseball way.

I far prefer Feynman's description of the same situation in *The Character of Physical Law* (MIT Press, 1965, page 128):

“We know how the electrons and light behave. But what can I call it? If I say they behave like particles I give the wrong impression; also if I say they behave like waves. They behave in their own inimitable way, which technically could be called the quantum mechanical way. They behave in a way that is like nothing you have ever seen before. Your experience with things that you have seen before is incomplete. The behavior of things on a very tiny scale is simply different. An atom does not behave like a weight hanging on a spring and oscillating. Nor does it behave like a miniature representation of the solar system with little planets going around in orbits. Nor is it like a cloud or fog of some sort surrounding the nucleus. It behaves like nothing you have seen before.”

The Meaning of “Explain” (Styer)

A few words about “explanation” or “understanding” in science. Sometimes a phenomenon can be explained in terms of something simpler. For example: “Why did it rain today?” “Because a cold front moved in.” “Well, why did a cold front move in.” “Because the jet stream pushed it.” “Well, why did the jet stream push it.” “Because the sun warmed Alberta and so deflected the jet stream”. “Well, why does sunlight warm objects?” And so forth. (Anyone who had raised a child knows such chains of questions all too well.) The point is that such chains get deeper and deeper to more and more fundamental topics, and at one point, they just stop. Why do photons behave as they do? We have a detailed theory, QED, that describes the way photons behave, but it doesn't explain why they behave that way. Explanation always involves going one layer deeper, and QED is the deepest we've got . . . for now. Maybe someday we'll have something deeper, but that won't fundamentally change the situation, because that will merely give another bottom layer that consists of description rather than explanation.

The Meaning of “Reality” (Taylor)

In my opinion, physics theory does not talk about “reality” directly. (Sorry!) Instead, theory helps you to discover what procedure YOU have to go through in order to make a prediction that can be verified by experiment. For example, QED tells you how to predict the probability that an electron will be detected at a give place and time. To help you visualize the procedure, quantum theory (I would claim EVERY scientific theory) uses metaphors, talking in this case about paths and clocks. Are these clocks “real”? Do their hands “really” rotate? Does the electron “really” follow all paths? As residents of Brooklyn say: “Fuggedaboddit” Forget about it. Asking such questions drives you crazy without helping in the practical prediction business. Of course, you can look for the electron on one of these paths, and use the same procedure to predict correctly the probability of detecting it there. But that is a *different* experiment and brings you no closer to “Reality.”

NOTE: Both “explain” and “reality” are discussed in much more detail—and deeply—in Chapter 2 of Steven Weinberg's *Dreams of a Final Theory* (Vintage, 1994, ISBN 0-679-74408-8)

Inside the Back Cover

SELECTED PHYSICAL CONSTANTS

Planck's constant	$h = 6.6261 \times 10^{-34}$ kilogram - meters ² /second
	$h \equiv \frac{h}{2\pi} = 1.0546 \times 10^{-34}$ kilogram - meter ² /second
Elementary charge	$e = 1.60218 \times 10^{-19}$ coulomb
Electron mass	$m_e = 9.109 \times 10^{-31}$ kilogram
Proton mass	$m_p = 1.67262 \times 10^{-27}$ kilogram

CONVERSION FACTOR

$$\text{joule} = 10^7 \text{ ergs}$$