QUANTUM STOPWATCH FOR THE FREE ELECTRON

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1. INTRODUCTION

FINALLY we get to the electron. Why does Feynman spend so long—more than half of his little book *QED* —on the photon? The answer may have three parts:

> **First,** emission and detection of light (photons) is part of our everyday lives, so Feynman can use our experience as background to introduce quantum behavior.
>
> **Second**, photons do not interact with one another under ordinary conditions. So a million photons (easy to see) will act according to the probabilities computed for a single photon (hard to see). In contrast, electrons are charged and repel one another. A million electrons bunched together in a vacuum act quite differently than a single electron.
>
> **Third,** what is weird about photons is also weird about electrons. Just about EVERYTHING in the strange behavior of photons has correspondence in the behavior of electrons—including stopwatch hands that rotate as an electron explores alternative paths, addition of arrows for each path to give a resulting arrow, and the squaring the magnitude of this resulting arrow to yield the probability of detection.

Of course, an electron is different from a photon. For one thing, the electron has mass, while the photon is massless. For another, as a consequence of the electron charge, the electron flies freely only in a vacuum. Also because of its charge, the electron is attracted to the nucleus and forms the wrapping of the atom. Most of the electrons in our immediate everyday surroundings are bound in atoms and molecules. One of these bundles of electrons and nuclei is your body!

But a central question remains, a question that Feynman does not answer in his little book:

How fast does the hand of the quantum stopwatch rotate for the electron?

2. CLUES FROM THE PHOTON

For the photon, we assume (correctly) that we can steal an answer to this question from the classical theory of light: The quantum stopwatch hand rotates once for each period of the classical wave. The higher the classical frequency, the faster the quantum stopwatch hand rotates. For blue light the quantum stopwatch hand rotates approximately twice as fast as for red light.

An electromagnetic wave can be defined and observed only when the beam contains a huge number of photons. The quantum stopwatch is far more fundamental than the classical wave. The stopwatch correctly describes the behavior of a single photon, the electromagnetic wave does not.

In contrast to the photon, there is no classical frequency associated with the electron. Classical mechanics as we usually learn it gives us no easy clue about the rate at which the quantum stopwatch rotates for the electron as it explores all the paths from source to detector. So we will try to use the photon as an example to find the result for the free electron.

Go back to a single “hunk” of electromagnetic energy—the photon. We know from the photoelectric effect and endless other evidence that the quantized energy E of the photon is related to the frequency f of the corresponding classical electromagnetic wave by the mongrel formula:

$$E = hf \qquad \text{(for the photon -- zero mass)} \qquad (1)$$

This formula is a “mongrel” because the quantized energy E on the left side of the equation comes from quantum mechanics, while the frequency f on the right side comes from classical electricity and magnetism. Here h is Planck’s constant, the quantity that always seems to appear, shouting "Caution!," whenever we pass from the classical world to the quantum world. For the photon we can rearrange the equation to give the frequency

$$f = \frac{E}{h} \qquad \text{(for the photon -- zero mass)} \qquad (2)$$

This is the frequency with which the stopwatch hand rotates (rotations/second) as the photon explores alternative paths.

3. QUANTUM STOPWATCH FOR THE FREE ELECTRON

Can we talk about a quantum clock for the electron? If so, what is the rate at which the hand of the quantum clock for the electron will rotate?

One can say that for a photon ALL energy is “kinetic”—when the photon stops, it is absorbed; it no longer exists. So we try a formula similar to equation (2) for a FREE electron, whose energy is also entirely kinetic. That is, we assume that the electron is not influenced by an electric field or a gravitational field or any other force or field. Then we write an equation similar to (2) for the electron:

$$f = \frac{KE}{h} = \frac{mv^2}{2h} \qquad \text{(for the electron or other massive particle)} \qquad (3)$$

It turns out that this equation is correct for the rotation rate of the clock hand for ANY FREE atomic particle that has mass, such as the electron, proton, or neutron. Even the results of experiments with whole atoms have been correctly predicted by equation (3). NOTICE that the frequency given by equation (3) increases with the mass of the particle and with the SQUARE of the velocity of the particle.

Q1. An electron moves with a speed of one meter per second. With what frequency f does the hand of its quantum clock rotate? (See inside the back cover of this workbook for physical constants.)

Q2. A ball has a mass of half a kilogram and moves with a speed of one meter per second. ASSUME that we can talk about the quantum clock of so large an object, and calculate the frequency f with which the hand of its quantum clock rotates. How many times as fast is this as the frequency for the electron in question Q1?

Q3. How many times as fast as the hand of the electron quantum clock does the hand of the proton quantum clock rotate, for the same velocity of each particle?

Q4. An electron moves with speed v_e. With what speed v_p must a proton move so that the hand of its quantum clock rotates at the same rate as that of the electron clock?

4. EXERCISES FOR THE FREE ELECTRON USING THE "ONE PARTICLE" PROGRAM

4A . BACKGROUND

The following exercises use the ELECTRON feature of the OneParticle program. For the electron display, the time between emission and detection is made the same for all paths, both short paths and long. Therefore the speed along the path is different for different paths. And so is the kinetic energy different for different paths. In principle, the speed along different segments of the *same* path could be different, yielding different kinetic energy along each segment. (More about this in later software programs.) However, in the OneParticle program the speed along *both* segments of a *given* path are the same (though this speed will be greater for longer paths than for shorter paths).

4B. INTRODUCTION DISPLAY

QUESTIONS 1 through 5 concern the INTRODUCTION display to the OneParticle for the ELECTRON. Simply start the program and choose Electron from the Particle menu at the top.

Q5. Comparing two paths. Construct a direct path between source and detector (with an intermediate point on the line between source and detector) and a second path much longer, with an intermediate point way off to one side. Looking at the display, choose: For which path, the longer or the shorter, does the clock rotate faster? (Remember that the time between emission and detection is fixed at the same value for all paths.)

Questions Q6 through Q10 are thought questions, for which the answers are NOT derived from using the computer display.

Q6. Why different clock rotation rates? Explain the reason for your observation in Q5 based on the description of the electron quantum clock in Section 3 above.

Q7. Special case: comparing speeds for two paths. Suppose that one path between source and detector is twice as long as another path. Assume that the speed is constant along both segments of each of these paths (but different on the two paths). Remember that the time between emission and detection is fixed at the same value for all paths. What is the ratio of speeds for the particle:
(speed on long path)/(speed on short path)?

Q8. Special case: comparing kinetic energies. For the case described in Q7, what is the ratio (kinetic energy on long path)/(kinetic energy on short path)?

Q9. Special case: comparing clock rotation rates. For the case described in Q7, what is the ratio (clock revolutions per second on long path)/(clock revolutions per second on short path)?

Q10 (Optional). Clock rotation per distance covered by particle. For the case described in Q7, what is the ratio (clock revolutions per DISTANCE traveled on long path)/(clock revolutions per DISTANCE traveled on short path)?

4C. MAIN DISPLAY

QUESTIONS Q11 and later concern the regular display, after the introduction with the ELECTRON chosen as particle. (See Particle menu at the top of the screen.)

Figure 1 shows a chain of eleven intermediate points between Source and Detector. These create five paths above the direct horizontal line between Source and Detector, five more paths below this line, and one along the direct line between Source and Detector.

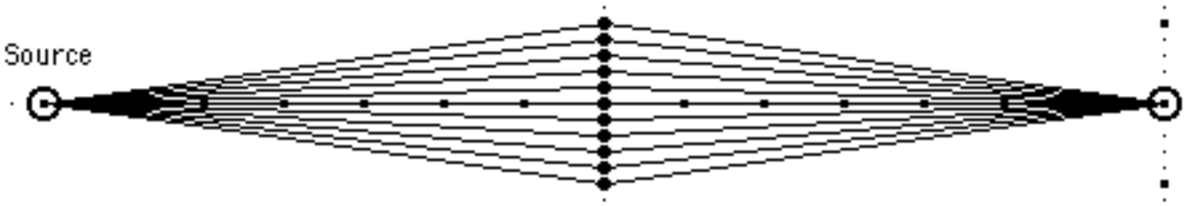

Figure 1: Clicking on a five dots along the vertical center line on either side of the horizontal direct path connecting Source and Detector (plus one dot ON this horizontal line).

THE CORNU SPIRAL

Q11. Reconstruct Figure 1, but this time creating a chain of intermediate points from the top of the window to the bottom. Use the Advanced menu, the button CHAIN OF MIDPOINTS, a click at the top of the screen and the button SYMMETRIC ACROSS HORIZONTAL CENTERLINE. Click on the RESULTING ARROW button and record the length of the resulting arrow, this number appearing at the lower left of the screen. NOTE that this length is only approximate, because adding (or subtracting) a single dot at one end of the chain can change the length of the resultant arrow by as much as the length (12 pixels) of one of the little arrows of which the Cornu spiral is composed in this display.

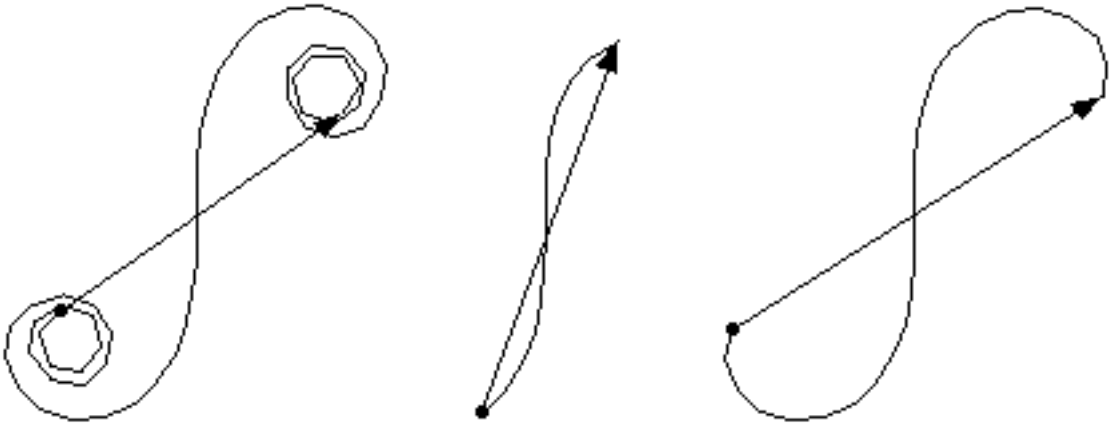

Figure 2. LEFT: A Cornu spiral and resulting arrow created with a chain of intermediate points extending from top to bottom of the screen of the OneParticle program. CENTER: A resulting arrow of approximately the same length using a limited number of dots on either side of the horizontal direct path between source and detector (similar to the case shown in Figure 1). RIGHT: A resulting arrow of approximately the same length using a greater number of intermediate dots. The right-hand arrow better approximates the direction as well as the length of the resultant arrow in the left-hand panel.

Q12. Using a method similar to that described in Q11, reconstruct a figure similar to Figure 1, but this time find by trial and error the SMALLEST number of dots on either side of the horizontal direct path that leads to a resulting arrow approximately the same length as the resulting arrow you measured in question Q11. Record the total number of dots and the length of the resulting arrow in pixels.

What is the goal here? We are trying to reproduce the resulting arrow of the left-hand panel in Figure 2 using a smaller vertical range of intermediate points. The arrow in the center panel is approximately of the correct length, but it points in a direction different from that of the left-hand arrow. However, we can make an alternative construction that yields a resulting arrow that not only has approximately the same length as the arrow at the left but also points in approximately the same direction. This is accomplished by adding more intermediate dots above and below the horizontal direct path, so that the Cornu S-curve "loops over" as shown in the right-hand panel of Figures 2 and 3.

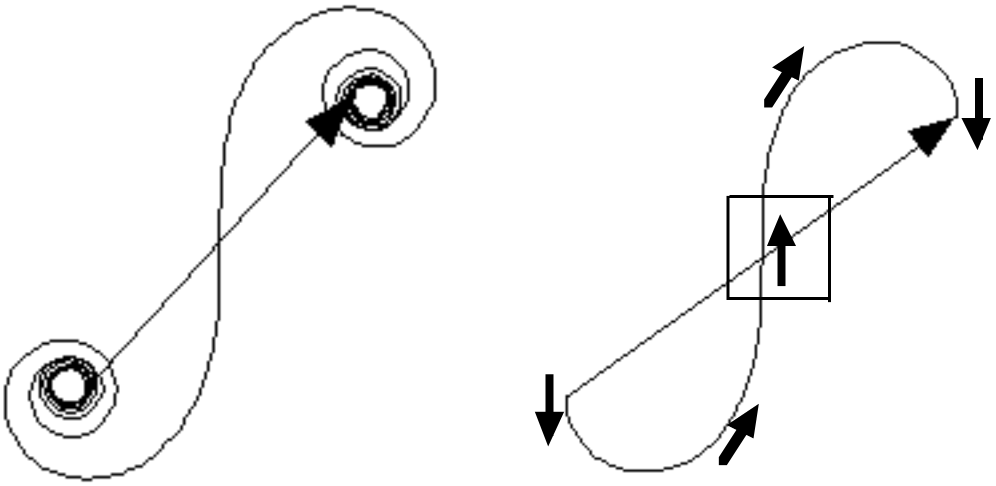

Figure 3. More detail of a construction, right, that uses a smaller vertical range of intermediate points than the (nearly) complete Cornu spiral at the left but gives a resulting arrow nearly the same in magnitude and direction. The simplified right-hand construction uses contributions from only those worldlines along which the number of rotations differs by one-half-rotation or less from the direct worldline. The boxed small fat black arrow in the middle of the right-hand diagram is from the direct worldline. Each fat arrow elsewhere shows the direction of the adjacent little arrow of which the curve is composed.

Q13. By trial and error find the number of chain-dots on either side of the horizontal direct path that leads to a figure similar to that in the right-hand panel of Figure 3 for which the resulting arrow is approximately the same length and direction as the one you constructed in Q11. (HINT: Across the center line, make a symmetric chain that is too long, then click-erase an equal number of dots on each side to satisfy the given conditions.) Report the total number of dots in the chain that leads to this result and the length of the resulting arrow.

COMMENT: Questions Q11 through Q13 may seem like Mickey Mouse exercises. The result, however, is *very* important. In Q13 you constructed a limited set of paths on either side of the direct path between Source and Detector. This limited set gives essentially the same resulting arrow as ALL worldlines with center points on the vertical dotted line between Source and Detector. Why is this? Because the little arrows from the paths above and below your limited set point in a variety of different directions and more or less cancel one other out. These little arrows from the paths on either side of your set are the ones that make up the tightly-wound "scrolls" on either end of the Cornu spiral in the left-hand panel of Figures 2 and 3.

INTERFERENCE PATTERNS FOR THE ELECTRON

Notice at the lower left of the screen that there are 10 rotations for the direct path between source and detector. Use the Advanced display (menu at top of original screen). Place and erase a series of dots along the vertical center line above the horizontal dotted line between source and detector to find additional points for which the number of rotations increase by integer values: 11, 12, 13 . . . , so that the little stopwatch hands at the right add up along approximately the same direction as the stopwatch hand for the direct path. Figure 2 shows the display part way through this process. Then click on the button REFLECT ACROSS CENTERLINE and click on the midpoint between source and detector to complete the set of “slits.”

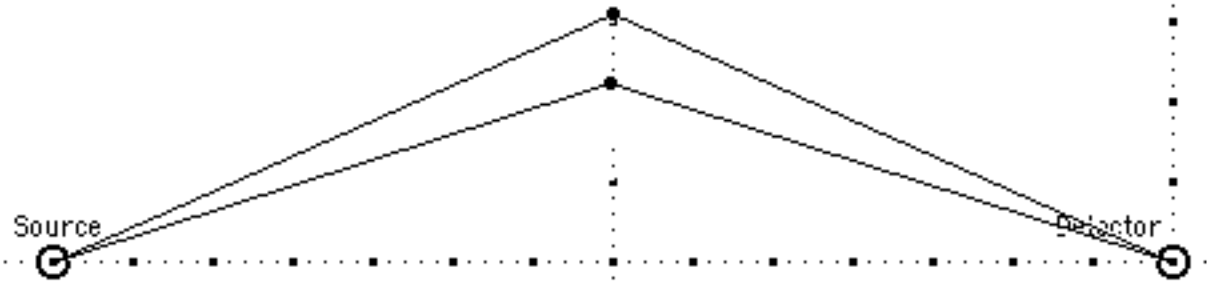

Figure 2. Finding paths whose stopwatch hands line up at the detector

Q14. Many-slit interference. What is the maximum number of DIFFERENT single-dot “slits” you can find ABOVE the mid-point that satisfy this criterion? To be specific, answer this question with the single NUMBER in the following sentence: “In addition to the central path, I find a maximum of (NUMBER) different slits ABOVE the central path within the display that give stopwatch hands that line up with the stopwatch hand from the central path.”

Q15. Multi-slit diffraction grating for the photon. Change the particle to RED PHOTON and answer the same question Q14 for the red photon.

Q16. Number of slits for photon and electron. There are approximately 10 rotations of the clock for the straight path in both electron and photon cases, Q14 and Q15. Are the number of slits you find in answer to Q14 and Q15 different for the photon than for the electron? If so, explain the difference in terms of the Sum Over Paths model used in each case.

Q17 (Optional). Spacing of slits. The “slits” in the ELECTRON interference grating are not equally spaced vertically. (The same is true of the corresponding “slits” in the PHOTON interference grating.) Explain why not.

TECHNICAL NOTE, MOSTLY FOR TEACHERS

The interference of electrons modeled above is quite different from electron interference in most pedagogic experiments. In the usual case, electrons from a source are given a nearly unique kinetic energy and therefore a unique frequency $f = KE/h$.

In contrast, our electron is emitted from a point in space and time (an emission *event*) with a wide spread in kinetic energies. This electron is detected with some probability at a particular later time and place (a detection *event*), for example by turning the detector on and then quickly off again. Therefore the electron has a fixed time to explore all paths from source to detector. Longer paths must be explored at higher velocity and thus with greater kinetic energy and greater rotation rate of the quantum clock, $f = KE/h$.

Why do we choose this alternative method of interference? Because it is consistent with the picture of quantum behavior we are composing: a particle emitted at one event in space and time and detected at a later event in space and time.