Feynman Diagrams

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. . . in the fall of 1940, Feynman received a telephone call from John Wheeler [Feynman's thesis advisor] at the Graduate College in Princeton, in which he [Wheeler] said that he knew why all electrons have the same charge and the same mass. "Why?" asked Feynman, and Wheeler replied, "Because they are all one and the same electron." [NOTE: Wheeler is saying that this single electron moves on many paths forward in time as an electron, each path snaking BACKWARD in time as a positron—see QED page 99. The multiple paths forward in time of this single electron represent all electrons. Hence there is only one electron in the universe—a perfect example of Wheeler's courage as a physicist!]. . . "But, Professor, there aren't as many positrons as electrons," and Wheeler replied, "Well, maybe they are hidden in the protons or something."

-- Jagdish Mehra

A. ELECTRONS AND THEIR INTERACTIONS

In Chapter 3 of the QED book, Feynman justifies some of the simplifying assumptions he made at the beginning of the book—such as assuming that photon partial reflection takes place at the surfaces of glass. Then he gives an outline of the deep theory of Quantum Electrodynamics (QED), for which he, Julian Schwinger, and Sin-Itiro Tomonaga jointly received the Nobel Prize in 1965. QED is decidedly relativistic in its outlook, for two reasons: (1) a major actor is the photon, which moves with the speed of light, and (2) particles are created and annihilated during the processes described by this theory.

Feynman introduces the great simplifying tool called the **Feynman diagram**. Using Feynman diagrams as a graphic computational aid, ordinary graduate students can keep track of all the possible ways that a system of particles can develop from a given initial state to a different final state. That is, they can account for all "paths," where we have expanded the meaning of the word "path" to include these Feynman diagrams.

Think of an electron. We know that it must explore all paths between fixed events of emission and detection. Now we add all possible SELF-INTERACTIONS that the electron can have along each of these worldlines, such as emitting a virtual photon which it later re-absorbs. (The phrase "virtual photon" means that the photon is not detected directly, but its fleeting creation and annihilation is inferred from its effect on the electron.) The theory that describes these added interactions is called Quantum Electrodynamics (QED). QED is an extension of the "explore all paths" theory we have been studying. And Feynman diagrams provide the tool for this extension.

Along with Feynman diagrams, you will meet the positron as an electron moving backward in time. This interpretation does seem to hold up, even though initially it led John Wheeler to believe incorrectly that there is only one electron in the universe. (See quote above.)

B. MOTION OF TWO ELECTRONS

(See two IMPORTANT notes at the end of question Q7, below.)

Questions Q1 thru Q7 refer to the two diagrams in Figure 59, page 93 of QED.
Assume an electron with no spin. Use the following values.

$x_1 = 2 \times 10^{-3}$ meters, $t_1 = 1 \times 10^{-3}$ seconds
$x_2 = 5 \times 10^{-3}$ meters, $t_2 = 1 \times 10^{-3}$ seconds
$x_3 = 1 \times 10^{-3}$ meters, $t_3 = 6 \times 10^{-3}$ seconds
$x_4 = 7 \times 10^{-3}$ meters, $t_4 = 7 \times 10^{-3}$ seconds

In order that we can compare answers, PLEASE ASSUME the following exact values:
mass of electron = exactly 9×10^{-31} kilogram
Planck's constant = h = exactly 6.6×10^{-34} Joule-second
ACCURACY: Please give all answers correct to three decimal places.

FIRST-WAY ARROW
Q1. How many times does the hand on the quantum clock rotate as the electron goes from event 1 to event 3? [HINT: Equation (3), page 25 of this Workbook is an expression for the rotation rate of the electron quantum clock hand, and the quantities given above tell for how long a time the hand of the quantum clock is rotating. Or you could use equation (3) of Workbook page 44 with $PE = 0$.]

Q2. How many times does the hand on the quantum clock rotate as the electron goes from event 2 to event 4?

Remember, when we multiply two amplitudes (two little quantum arrows: QED, page 61), the result has the magnitude given by the PRODUCT of the magnitudes of the two amplitudes, and the angle is the SUM of the two angles. ASSUME that here the magnitude of both elementary amplitudes is equal to one.

Q3. What is the resulting rotation of the product: E(1 to 3)*E(2 to 4)? This gives us the direction of the "first way arrow" Feynman mentions just above Figure 59, page 93 of QED.

SECOND-WAY ARROW
Q4. Answer question Q1 for the electron going from event 1 to event 4.

Q5. Answer question Q2 for the electron going from event 2 to event 3.

Q6. Answer question Q3 for the product E(1 to 4)*E(2 to 3). This gives us the direction of the "second way arrow" Feynman mentions below Figure 59, page 93 of QED.

RESULTING ARROW

Q7. Now add the two arrows calculated in Q3 and Q6. This means add the little arrows AS ARROWS. Remember the magnitude of each such little arrow is unity. What is the APPROXIMATE value of the length of the arrow that results from this summation?

TWO IMPORTANT NOTES:

NOTE #1: In analyzing the left-hand Figure 59, page 93 (for example), we multiply the amplitudes for the concomitant events: E(1 to 4)*E(2 to 3). This multiplication was originally explained on QED page 72. In QM we multiply the AMPLITUDES. By contrast, in classical physics we multiply PROBABILITIES. Suppose Bill Emmons flips a coin in Anchorage, Alaska and Carlos Gunter flips a coin in New Orleans, Louisiana. These flips are independent—we say they are "concomitant." Each has a probability of 1/2 that the coin will come up heads. The probability that BOTH coins will come up heads is (1/2)*(1/2) = 1/4, a multiplication.

[This leads to the joke about the statistician who always carries a bomb onto the airplane whenever he flies. His reason: The chance of there being TWO bombs on an airplane is vanishingly small, since it is the PRODUCT of two very small probabilities.]

In QM, by contrast, we multiply not the probabilities but rather the AMPLITUDES. Then later when we want to reckon the probability we take the SQUARED MAGNITUDE of the final amplitude. (Arrows can cancel, probabilities cannot—THIS is the great difference between classical physics and quantum physics.)

NOTE #2: The two sources 1 and 2 in Figure 59 are independent. This means that the INITIAL STARTING angle of the two quantum clock hands are probably NOT the same. The clock for emission 1 may start at zero (straight up), while the clock for emission 2 may start at ninety degrees from straight up. We say the two sources are INCOHERENT. Does this ruin our analysis above? No. The reason is that this just adds a common angle ϕ to the two amplitudes E(1 to 3)*E(2 to 4) and E(1 to 4)*E(2 to 3). But COMMON angles do not matter when we take the squared magnitude of the sum in order to find the probability.

More technically, in terms of complex numbers (page 47 of this Workbook), the common angle is the same as multiplying each of the product amplitudes by $e^{i\phi}$ which can also be written exp(iϕ). This common factor can be factored out of the sum of amplitudes. Its magnitude is unity, so its squared magnitude does not change the probability.
(END of Notes #1 and #2.)

C. ALTERNATIVE TWO-ELECTRON PATHS

Question Q8 refers to Figure 60, page 94 of QED. Look at the LEFT-hand diagram in that figure and at the expression for the corresponding amplitude given in the third and fourth lines from the bottom of page 94.

Q8. Write down the corresponding expression for the amplitude for the RIGHT-hand diagram in Figure 60.

D. MOTION OF A PHOTON THROUGH GLASS

Questions Q9 thru Q14 refer to Figure 68, page 104 of QED:

Q9. All six worldlines arrive at detection event A at the same time. But travel along all six worldlines occurs at the same speed—the speed of light. Then how can it be that the corresponding six emission events are at different times?

Q10. For glass, the six small arrows shown in Fig. 68, page 104, when added, give approximately a quarter of a circle (angle of 90 degrees) of radius 0.2. What is the length of the resulting arrow?

Q11. What is the probability of detection for this thickness of glass?

Q12. How many MORE arrows (corresponding to an increased thickness of glass) would one have to add in to get a total amplitude of zero.

Q13. How does this new thickness of glass for zero reflection compare with that shown in the figure? Is it the same thickness? half as thick? twice as thick? . . . as thick?

Q14. In general, does the length of the resultant arrow for reflection depend on the thickness of the glass?

D. GROUPING OF PHOTONS

NOTE: For Q15 thru Q17: simply write your answer in terms of the notation found in the reading. No numerical answers are requested unless the result is zero.

Q15. A photon is emitted at event 1 and another photon is emitted at event 2 (Figure 71, page 111). Write down the resultant AMPLITUDE for BOTH photons to reach event 3? Express this resultant amplitude in terms of the amplitudes P(1 to 3) and P(2 to 3). (For notation, see the QED reading.)

Q16. Write down the expression for the PROBABILITY that the two photons will arrive at event 3 using the amplitude derived in Q15.

E. AVOIDANCE OF ELECTRONS

Q17. An electron is emitted at event 1 and another electron is emitted at event 2 (Figure 72, page 112). Write down the resultant AMPLITUDE for BOTH of them to reach event 3? Express this resultant amplitude in terms of the amplitudes E(1 to 3) and E(2 to 3)

Q18. Write down the expression for the PROBABILITY that the two electrons will arrive at event 3 using the amplitude derived in Q17.

F. FURTHER FEYNMAN DIAGRAMS

Q19. Figure 75, page 117, shows Feynman diagrams, each of which includes FOUR extra "couplings." that is four extra intersection points beyond the one shown in the original diagram, Figure 73. Figure 76, page 118 shows a few examples of Feynman diagrams that include SIX extra couplings. Why is there no intermediate figure showing Feynman diagrams that include FIVE extra couplings?

Q20. Suppose you are given each of the amplitudes computed from the Feynman diagrams in figures 73, 74, and 75 for an electron to go from event 1 to event 2 while absorbing an incoming photon in the process. How would you use these individual amplitudes to find the total amplitude for the process, to an accuracy of "four extra couplings" (Figure 75)?

Q21. Why does the computation described in Q20 lead to a very accurate answer, even though we ignore the more complicated processes ("six extra couplings," "eight extra couplings," etc.)

Q22. Why have the amplitudes not been calculated for the more complicated processes mentioned in Q21 (even aside from the question of accurate knowledge of the coupling constant *j*)?

REFERENCE for quote at beginning of this section:
Jagdish Mehra, *The Beat of a Different Drum, The Life and Science of Richard Feynman,* Oxford, 1994, pages 113-115.

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