Exploring Black Holes

Introduction to General Relativity

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Can I see a black hole at all? If I can see it, what does a black hole look like? Does it *look* black? How big does a black hole look when I am at rest nearby? Does it look different when I fall toward it? What does it *feel* like to fall toward a black hole? Am I comfortable? Do I see stars overhead as I fall into a black hole? If so, do these stars change position or color? Can I receive messages and packages from my friends on the outside? Is it true that I cannot send anything to my friends outside, not even a light signal? Why not? How long do I live once I fall into a black hole? Will I reach the center, or will something happen to me on the way? If I reach the center, can I see the center as I approach? Why do I die at the center—in what way does my body stop working? Is my death quick and painless? What is the last thing I see? Is all matter crushed to a point at the center of a black hole? Where do black holes exist: at centers of galaxies or alone in intergalactic space? How does the orbit of a stone around a black hole differ from the orbit of a planet around our Sun? How fast does the stone move in a circular orbit around a black hole? Does the stone's speed reach the speed of light? How close to a black hole can I move in a circular orbit? How can I use orbits around a black hole to travel forward in time? Can I use a black hole to travel backward in time? Does light have its usual speed near a black hole? In what sense is spacetime a unity? Does the term *relativity* mean that everything is relative? What does "curvature of spacetime" *mean*? How can I observe this curvature? How does the Global Positioning System work? Is general relativity important to its operation? Does light "get tired" as it moves away from Earth? away from a black hole? How can I observe this tiredness? Does light change direction as it passes our Sun? Does its direction change a lot? Does the change in direction depend on the color of the light? How do I observe this change in direction? How does an astronomical object focus light from a distant star or galaxy? What does this focused light look like? Can light go into orbit around a black hole? Does the mass of a stone change with its velocity? On Earth how do I change mass into energy? energy into mass? How do I use a black hole to convert mass to energy? energy to mass? Does a stone plunging into a black hole reach the speed of light? Does the plunging stone move faster than light? What happens to the mass of a black hole when it swallows a plunging stone? How great is the acceleration of gravity near a black hole? Can a black hole spin? If so, what is the fastest it can spin? Does a spinning black hole drag space around with it? What does "dragging space" *mean*? How do I observe this dragging? What happens to light that moves with or against the rotation? How can I harness the rotational energy of a spinning black hole to power my machines? Do spinning black holes power quasars? What's a quasar? Can I use a spinning black hole for hyperspace travel to other parts of the Universe? Is the Universe just a very big black hole? How does the size of the Universe change with time? What does this change in size *mean*? Will the Universe recontract to a Big Crunch? Or will the Universe expand forever? Do recent observations predict the final fate of the Universe?

Curiosity, like coffee, is an acquired need. Just a titillation
at the beginning, it becomes with training a raging passion.

— Nicholas S. Thompson

Acknowledgments

For more than 12 years, students in many classes have read through sequential versions of this text, shared with us their detailed difficulties, and given us advice. (See "Only the Student Knows," *American Journal of Physics*, Volume 60, Number 3, March 1992, pages 201–202.) Many of the "thinker objections" in the text come from these commentators.

Versions of the book were used in classes taught online over the National Teachers Enhancement Network out of Montana State University. Many class members were high school science teachers who brought their professional skill and interest to the educational effectiveness as well as to the physics content of the book.

Nora Thornber has taught several online classes using versions of this book, meticulously read endless drafts, offered a host of suggestions, and improved and simplified a large number of the derivations. She has spent more time and effort on this book than anyone but the authors.

A number of relativity specialists—graduate students, postdoctoral fellows, faculty members, and others—examined parts or all of various versions of this book and made comments that increased clarity, introduced modern results, and corrected many errors. In alphabetical order they are Kashif Alvi, Edmund Bertschinger, Lior M. Burko, Sean M. Carroll, Teviet D. Creighton, Alexei V. Filippenko, Sergio Goncalves, Tom Goodale, William A. Hiscock, Lawrence E. Kidder, Shane L. Larson, Lee Lindblom, Yuk-Tung Liu, Anupam Mazumdar, Jonathan Morris, Mark A. Pelath, Patricia Purdue, Yuri Levin, Kip S. Thorne, Michele Vallisneri, Vrata Venet, Clifford M. Will, and Kazunori Yoshida. The authors regret that they cannot blame remaining errors on this august company.

G. P. Sastry offered much detailed and general advice on the book at several stages and supervised 14 students who read a late version, provided comments, and solved all exercises. He and two students, Manish Niranjan and Saswat Sarangi, turned their solutions into a Solutions Manual for this book.

Philip Morrison made several suggestions and convinced us not to invoke that weird quantum particle, the photon, in a treatment of the classical theory of relativity (except in some exercises).

Eric Sheldon read many drafts of this book; his good judgment and wide acquaintance with the literature have clarified and enriched the presentation. Sanjoy Mahajan significantly improved both the physics and the clarity of expression. Ira Mantz offered a wide spectrum of careful, critical, and useful comments on late versions. Stamatis Vokos and Rachel E. Scherr taught from earlier drafts and offered many helpful suggestions. Rachel Scherr championed inclusion of the questions inside the front cover and revised these questions. Beta Keramati recommended placing questions at the beginnings of chapters and projects. Ewa M. Basinska located many of the references. Peter M. Brown made many suggestions, found quotations, drafted the Glossary of Terms, helped to assemble the reading list, and had the initial idea for the front cover. Michael J. Tsai originated and helped to develop the back cover design. Other reviewers of various drafts: Curt Covey Sally Ganner, Roy Gould, Vivek Iyer, Roy Lisker, Sergey V. Siparov. Penny Hull's copyediting improved the clarity and simplicity of both the English and the physics.

And then there is Carla . . .

Addison Wesley Longman, publisher 'n stuff about AWL

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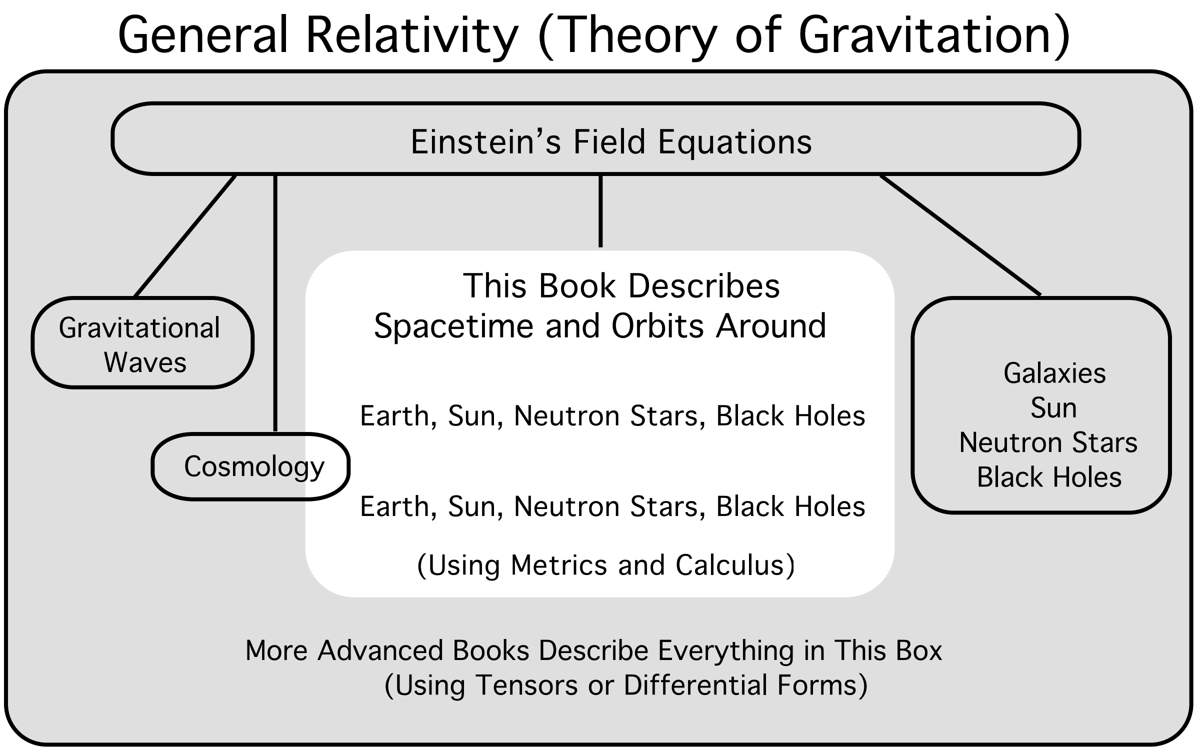
Key words: 1. General relativity. 2. Black Hole. 3. Relativity. 4. Cosmology. 5. Spinning black hole.

Note: *Both females and males make competent observers. Ordinarily we treat as female any observer in free float (whether in orbit or plunging inward) as well as the remote observers called Schwarzschild and Kerr bookkeepers. (See the Glossary of Terms for definitions.) Male observers stand on spherical shells around a nonrotating black hole and ride on circular rings around a spinning black hole. In other settings we try to use gender-neutral language. Where this is not convenient we usually assign gender randomly.*



Exploring Black Holes

General relativity flows from Einstein's field equations, which relate the mass and pressure in a region of spacetime to the "warping" of spacetime across that region. The field equations reveal how this warping is experienced by any observer, whether she is moving, accelerating, rotating, stretching, twisting, or tumbling. The field equations are wonderfully general. But this generality has a price: mathematical sophistication. The field equations speak the language of tensors or differential forms, which closes off this fascinating subject to some people and delays the involvement of others.

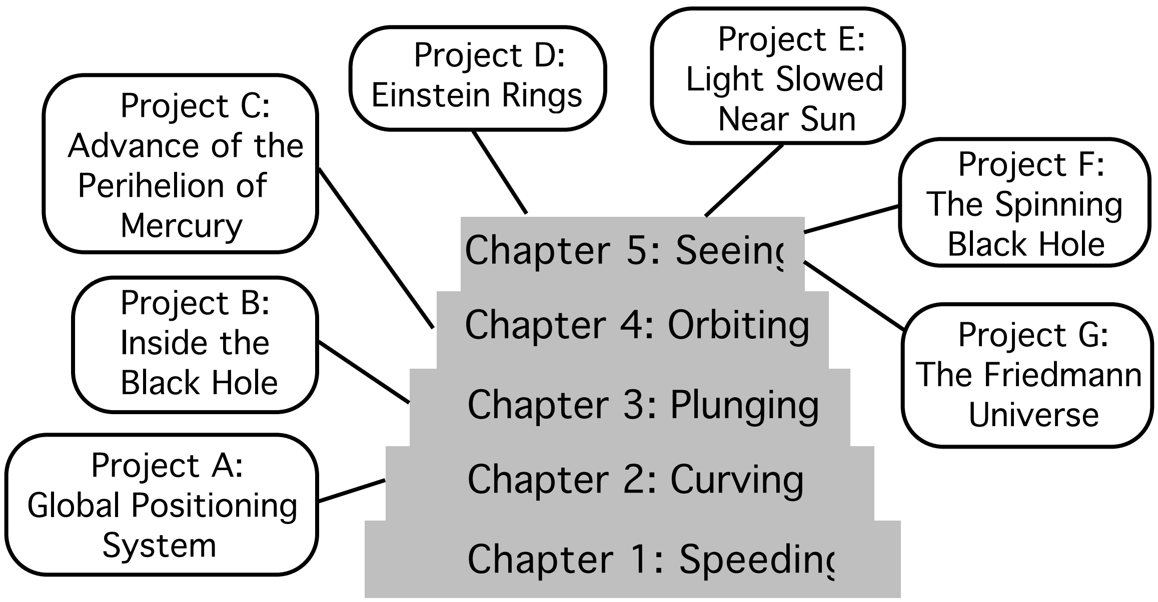
This book does not start with the field equations but rather with the primary solutions of these equations: the so-called **metrics** that describe curved spacetime around nonspinning and spinning centers of gravitational attraction.

The metric helps to answer every scientific question about (nonquantum) features of spacetime surrounding a black hole, every possible question about trajectories of light and satellites around the black hole as well as around more familiar centers of attraction such as Earth and Sun. The metric for a rotating black hole may tell us about quasars, the most powerful steady energy sources in the Universe. The black-hole metric brings preliminary insights about the history and structure of the Cosmos.

Using the metric requires only algebra, elementary differential calculus, and a handful of integrals. This modest mathematics opens the subject to the interested person and paves the way to a deeper study of general relativity for one who will discover new truth about this strange and beautiful Universe, our home.

Key idea: Spacetime tells mass how to move; mass tells spacetime how to curve.

Overview



CHAPTERS provide the reader with background needed to carry out exercises and projects.

Chapter 1 Speeding Key ideas from special relativity that are useful in general relativity. We meet the metric for flat spacetime.

Chapter 2 Curving The Schwarzschild metric describes the curvature of spacetime near a nonrotating Earth, Sun, neutron star, or black hole.

Chapter 3 Plunging A stone plunging radially exhibits a constant of the motion: energy.

Chapter 4 Orbiting Orbits of stone and planet derive from two constants of the motion: energy and angular momentum. Predict the shape of an orbit at a glance.

Chapter 5 Seeing What you see when you look at, around, and outward from a black hole. Your view as you plunge through the horizon and approach the crunch point at the center of a black hole.

PROJECTS help the reader explore a topic, fill in the steps, compute physical outcomes, and carry out his or her own investigations.

Project A Global Positioning System
General relativity is crucial to its operation.

Project B Inside the Black Hole A one-way trip to the crunch point at the center.

Project C Advance of the Perihelion of Mercury Small changes in the orbit of Mercury showed Einstein that his new general relativity theory was correct.

Project D Einstein Rings Light deflected by dark galactic objects helps us to see them.

Project E Light Slowed Near Sun Light moves slower near Sun. How do we know?

Project F The Spinning Black Hole "Frame dragging" near Earth or black hole.

Project G The Friedmann Universe Simplest model of the evolving Cosmos. Is it correct?

PREFACE

A SINGLE GOAL

This book, Exploring Black Holes, makes a quick, directed thrust through general relativity and black holes. It includes many topics, all with a single goal.

THE GOAL: Power to the Reader! *We provide tools to answer questions and carry out calculations about curved spacetime near Earth and black holes. Topics are limited to those in which you can be an active participant, starting with an elementary knowledge of calculus and special relativity. Tools developed in general relativity then help you to pursue your own investigations.*

—Edwin F. Taylor and John Archibald Wheeler

QUESTIONS NOT ANSWERED AND WHAT TO DO ABOUT THEM

Here are some important questions *not* answered in this book.

- What is the full range of phenomena covered by general relativity?
- How did general relativity begin, and who wrestled with the ideas presented in this book?
- What about gravitational waves, pulsars, supernovas, and the formation of stars and black holes?
- Where are the frontiers of the subject, and how far can these frontiers be pushed?
- What are the latest results from observational satellites and the latest theories about the origin of the Universe?

To find the latest observational results, no source is better than the World Wide Web, on which addresses change from month to month, even from hour to hour. Here is a website of Hubble pictures and commentary: http://oposite. stsci.edu/pubinfo/pictures.html. For more current information, ask your 15-year-old consultant.

To help you engage some of the other questions *not* answered in this book, we know of no better popular source than the following book, which is an almost perfect complement to *Exploring Black Holes*:

Black Holes and Time Warps
Einstein's Outrageous Legacy
Kip S. Thorne
W. W. Norton, New York, 1994
ISBN 0-393-31276-3 paperback
Phone order: 1-800-233-4830

Six hundred pages and 0.9 kilogram of history, people, theory, results, and speculation. It is hard to imagine a more complete, engrossing, or enjoyable survey by a single major participant. The book is thoroughly cross-referenced with notes, list of characters, chronology, glossary, people index, and subject index. We suggest that you start Thorne’s book with the Prologue: A Voyage among the Holes, in which the reader, in a science fiction tale, encounters black holes and their strange properties.

One author (EFT) will attempt to provide updates for *Exploring Black Holes* on the website http://www.eftaylor.com/

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