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Quantum physics explains Newton's laws of motion

Jon Ogborn[1] **and Edwin F Taylor**[2]

[1] Institute of Education, University of London, UK
[2] Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract
Newton was obliged to give his laws of motion as fundamental axioms. But today we know that the quantum world is fundamental, and Newton's laws can be seen as *consequences* of fundamental quantum laws. This article traces this transition from fundamental quantum mechanics to derived classical mechanics.

Explaining

It is common to present quantum physics and the behaviour of quantum objects such as electrons or photons as mysterious and peculiar. And indeed, in Richard Feynman's words, electrons do 'behave in their own inimitable way . . . in a way that is like nothing that you have ever seen before.' (Feynman 1965, p 128).

But it was also Richard Feynman who devised a way to describe quantum behaviour with astounding simplicity and clarity (Feynman 1985). There is no longer any need for the mystery that comes from trying to describe quantum behaviour as some strange approximation to the classical behaviour of waves and particles. Instead we turn the job of explaining around. We *start from* quantum behaviour and show how this *explains* classical behaviour.

This may make you uncomfortable at first. Why explain familiar things in terms of something unfamiliar? But this is the way explanations *have* to work. Explanations that don't start somewhere else than what they explain don't explain!

In this article we show that quantum mechanics actually *explains* why Newton's laws of motion are good enough to predict how footballs and satellites move. For Newton, fundamental laws had to be axioms—starting points. For us, Newton's laws are seen to be *consequences* of the fundamental way the quantum world works.

Fermat's principle: source of the key quantum idea

Geometrical optics predicts the formation of images by light rays. Examples are images due to reflection and images formed by eyeglasses and camera lenses. All of geometrical optics—every path of every light ray—can be predicted from a single principle: *Between source and reception point light travels along a path that takes the shortest possible time*. This is called *Fermat's principle* after the Frenchman Pierre de Fermat (1601–1665).

A simple example of Fermat's principle is the law of reflection: angle of incidence equals angle of reflection. Fermat's principle also accounts for the action of a lens: a lens places different thicknesses of glass along different paths so that every ray takes the *same time* to travel from a point on the source to the corresponding point on the image. Fermat's principle accounts for how a curved mirror in a telescope works: the mirror is bent so that each path takes the *same time* to reach the focus. These and other examples are discussed in the *Advancing Physics* AS Student's Book (Ogborn and Whitehouse 2000).

Fermat's contemporaries had a fundamental objection to his principle, asking him a profound question, 'How could the light possibly know in advance which path is the quickest?' The answer

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goes very deep and was delivered fully only in the twentieth century. Here is the key idea: *The light explores all possible paths between emission and reception*. Later we will find a similar rule for motion of atomic particles such as the electron: a particle explores *all possible paths* between source and detector. This is the basic idea behind the formulation of quantum mechanics developed by Richard Feynman (1985).

The idea of exploring all possible paths raises two deep questions: (1) What does it *mean* to explore all paths? (2) How can 'explore all paths' be reconciled with the fact that everyday objects (such as footballs) and light rays follow unique single paths? To answer these questions is to understand the bridge that connects quantum mechanics to Newton's mechanics.

What does *Explore all paths!* mean?

The idea of exploring all paths descends from Christiaan Huygens' idea of wavelets (1690). Huygens explained the propagation of a wavefront by imagining that each point on the wavefront sends out a spherical wavelet. He could then show that the wavelets reconstitute the wavefront at a later time; the parts of the wavelets going everywhere else just cancel each other out. In 1819 the French road and bridge engineer Augustin Fresnel put the idea on a sound mathematical basis and used it to explain optical diffraction and interference effects in precise detail.

In the 1940s Richard Feynman (following a hint from Dirac) adapted Huygens' idea to give quantum physics a new foundation, starting with the quantum of light: the *photon*. Nature's simple three-word command to the photon is *Explore all paths!*; try every possible route from source to detector. Each possible path is associated with a change of phase. One can imagine a photon having a 'stopclock' whose hand rotates at the classical frequency of the light. The rotation starts when the photon is emitted; the rotation stops when the photon arrives at the detector. The final position of the hand gives an 'arrow' for that path.

The photon explores all paths between emission event and a possible detection event. The arrows for all paths are to be added head-to-tail (that is, taking account of their phases, just as wavelets are to be superposed) to find the total resultant quantum amplitude (resultant arrow) for an event. This resultant arrow describes the emission of a photon at one place and time and its detection at a different place and time. (There are also rules for how the length of the arrow changes with distance, which yield an inverse square law of intensity with distance. For simplicity we consider only cases where the distances vary little and changes in arrow length can be ignored.)

The resultant arrow determines the probability of the event. The probability is equal to the (suitably normalized) square of the length of the arrow. In this way the classical result that the intensity is proportional to the square of the wave amplitude is recovered.

We have outlined Feynman's simple and vivid description of 'quantum behaviour' for a photon. In effect he steals the mathematics of Huygens' wavelets without assuming that there are waves. For Huygens, wavelets go everywhere because that is what waves do. For Feynman, photons 'go everywhere' because that is what photons do.

The bridge from quantum to classical physics

The next question is this: How can Nature's command *Explore all paths!* be made to fit with our everyday observation that an object such as a football or a light ray follows *a single path*?

The short answer is *It does* ***not*** *follow a single path!* There is no clean limit between particles that can be shown to explore many paths and everyday objects. What do you mean by *everyday objects* anyway? Things with a wide range of masses and structural complexity are 'everyday' objects of study by many scientists. A recent example is interference observed for the large molecules of the fullerene carbon-70, which has the approximate mass of 840 protons (www.quantum.at).

In fact quantum behaviour tapers off gradually into classical behaviour. This and the following sections show you how to predict the range of this taper for various kinds of observations. In the meantime, as you look around you, think about the deep sense in which the football goes from foot to goal by way of Japan. So do the photons by which you see your nearby friend!

The key idea is illustrated in figure 1 for the case of photons reflected from a mirror. The mirror is conceptually divided into little segments, sub-mirrors labelled A to M. The little arrows shown under each section in the middle panel correspond

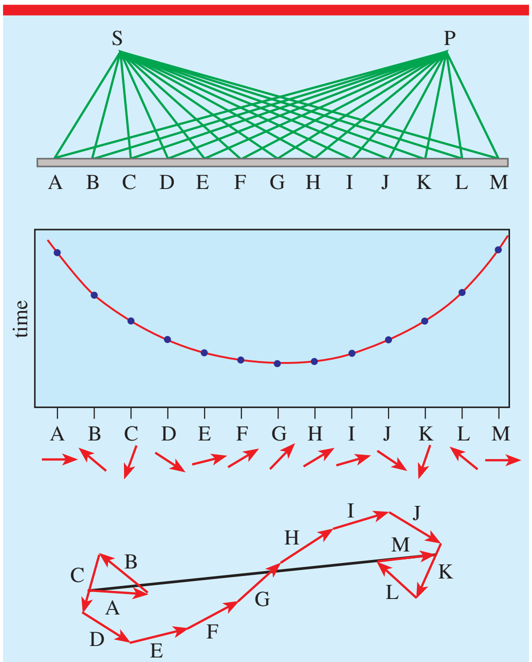

Figure 1. Many paths account of reflection at a mirrror (adapted from Feynman 1985, p 43). Each path the light could go (in this simplified situation) is shown at the top, with a point on the graph below showing the time it takes a photon to go from the source to that point on the mirror, and then to the photomultiplier. Below the graph is the direction of each arow, and at the bottom is the result of adding all the arrows. It is evident that the major contribution to the final arrow's length is made by arrows E through I, whose directions are nearly the same because the timing of their paths is nearly the same. This also happens to be where the total time is least. It is therefore approximately right to say that light goes where the time is least.

to the hand of the stopclock when the little ray from that section arrives at the point of observation P. In the bottom panel these little arrows are added up head-to-tail[1] in order to predict the resulting large arrow at point P. It is the squared magnitude of this resulting arrow that determines the probability that the photon will arrive at P.

As the caption to the figure comments, the arrows E to I make the greatest contribution to the final arrow because their directions are almost the same as one another. Arrows from nearby mirror segments on either end of the mirror point in many directions, so total contributions from these end-segments never amount to much. Even if we extend the mirror AM on each side to make it longer, the contributions to the resulting arrow made by reflections from the sections of these right-and-left extensions add almost nothing to the total resulting arrow. Why? Because they will all curl even more tightly than arrows ABC and KLM at the two ends of the resulting arrow shown in the bottom panel of the figure. Most of the resulting arrow comes from the small proportion of arrows that 'line up', and almost nothing comes from those that 'curl up'.

Now you see how *Explore all paths!* leads to a narrow spread of paths that contribute significantly to the resulting arrow at P. And that narrow spread must lie near the path of minimum time of travel, because that is where the time, and so the phase, varies only slightly from path to nearby path.

Here then is the answer to Fermat's critics. They asked 'How could the light possibly know in advance which path is the quickest?' Answer: the photon does *not* know in advance: it explores all paths. However, *only* paths nearest to the quickest path contribute significantly to the resulting arrow and therefore to its squared magnitude, the probability that the photon will arrive at any point P.

This answer delivers more than a crushing riposte. It goes further and tells by how much Fermat's prescription can be in error. How big is the spread of paths around the single classical path? To give as wide a range as possible to the paths that contribute to the resulting arrow, find the arrows nearest to the centre that point in nearly the opposite direction to the central arrows G and H in figure 1. Little arrows C and K point in nearly the opposite direction to G and H. So our generous criterion for contribution to the resulting arrow is the following:

> Find the little arrows that point most nearly in the direction of the resulting arrow. Call these the central arrows. The range of arrows around these central arrows that contribute most significantly to the resulting arrow are those which point less than half a revolution away from the direction of the central arrows.

As an example, think of viewing a source of light through a slit, as shown in figure 2. Limit

[1] This method of adding up contributions was invented by Fresnel. If we make the mirror very wide and divide it into thousands of much smaller segments, the resulting plot of combined arrows becomes smooth, and is known as Cornu's spiral.

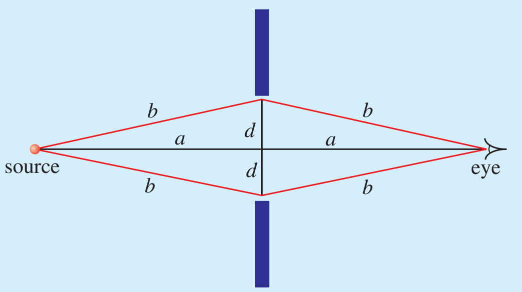

Figure 2. Extreme paths through a slit (not to scale).

the infinite number of possible paths to those consisting of two straight segments of equal length between source and eye. How wide (width $2d$ in the figure) does the slit have to be in order to pass most of the light from the source that we would observe by eye?

We can get numbers quickly. Apply Pythagoras' theorem to one of the right triangles abd in the figure:

$$a^2 + d^2 = b^2$$

or

$$d^2 = b^2 - a^2 = (b+a)(b-a) \approx 2a(b-a).$$

In the last step we have made the assumption that a and b are nearly the same length; that is, we make a small percentage error by equating $(b+a)$ to $2a$. We will check this assumption after substituting numerical values.

Our criterion is that the difference $2(b-a)$ between the paths be equal to half a wavelength, the distance over which the stopclock hand reverses direction. Take the distance a between slit and either source or receptor to be $a = 1$ metre and use green light for which $\lambda = 600\,\text{nm} = 60 \times 10^{-8}$ metres. Then

$$d^2 \approx a\frac{\lambda}{2} = 1 \times 30 \times 10^{-8}\ \text{m}^2 \qquad (1)$$

so that d is about 5×10^{-4} m. Therefore $2d$, the width of the slit, is about one millimetre. (Check: $b^2 = a^2 + d^2 = (1 + 3 \times 10^{-7})\ \text{m}^2$, so our assumption that $b + a \approx 2a$ is justified.)[2]

[2] Try looking at a nearby bright object through the slit formed by two fingers held parallel and close together at arm's length. At a finger separation of a millimetre or so, the object looks just as bright. When the gap between fingers closes up, the image spreads; the result of *diffraction*. For a very narrow range of alternative paths, geometrical optics and Fermat's principle no longer rule. But arrow-adding still works.

Electrons do it too!

In our analyses of photon reflection and straight-line transmission we assumed that the hand of the photon stopclock rotates at the frequency f of the classical wave. If we are to use a similar analysis for an electron or other 'ordinary' submicroscopic particle, we need to know the corresponding frequency of rotation of its quantum stopclock.

With this question we have reached bottom: there is nothing more fundamental with which to answer this question than simply to give the answer that underlies nonrelativistic quantum mechanics. This answer forms Feynman's new basis for quantum physics, then propagates upward, forming the bridge by which quantum mechanics explains Newtonian mechanics.

Here then is that fundamental answer: For an 'ordinary' particle, a particle with mass, the quantum stopclock rotates at a frequency

$$f = \frac{L}{h} = \frac{K - U}{h} \qquad \text{(nonrelativistic particles).} \qquad (2)$$

In this equation h is the famous quantum of action known as the Planck constant. L is called the Lagrangian. For single particles moving at nonrelativistic speeds the Lagrangian is given by the difference between the kinetic energy K and the potential energy U.

Is this weird? *Of course* it is weird. Remember: 'Explanations that don't start somewhere else than what they explain don't explain!' If equation (2) for a particle were not weird, the ancients would have discovered it. The ancients were just as smart as we are, but they had not experienced the long, slow development of physical theory needed to arrive at this equation. Equation (2) is the kernel of how the microworld works: accept it; celebrate it!

For a free electron equation (2) reduces to

$$f = \frac{K}{h} \qquad \text{(nonrelativistic free particle).} \qquad (3)$$

This looks a lot like the corresponding equation for photons:

$$f = \frac{E}{h} \qquad \text{(photon)}$$

(though mere likeness proves nothing, of course).

With the fundamental assumption of equation (2), all the analysis above concerning the photon can be translated into a description of the behaviour of the electron. We can ask, 'For what speed does the free electron have the same frequency as green light?' Green light has frequency $f = c/\lambda = 0.5 \times 10^{15}$ Hz. From equation (3) you can show that the speed v of such an electron is about one-tenth of the speed of light, near the boundary between nonrelativistic and relativistic phenomena. The energy of this electron is approximately 3×10^{-16} J or 2000 eV, a modest accelerating voltage. For a proton or hydrogen atom the mass is about 2000 times greater and the speed is less by the square root of this, a factor of about 1/45. For the carbon-70 molecule mentioned previously, the speed is less than a kilometre per second to get a quantum frequency equal to that of green light.

For particles of greater mass the quantum frequency in equation (3) increases and the effective wavelength decreases. Because the numerical magnitude of the quantum of action h is so small, the range of trajectories like those in figure 2 contracts rapidly, for particles of increasing mass, toward the single trajectory we observe in everyday life.

It is possible to compare the *Try all paths!* story to a wave story, and identify a 'wavelength' for an electron (see the Appendix). The result is the well-known de Broglie relation:

$$\lambda = \frac{h}{mv}. \qquad (4)$$

This lets us estimate the quantum spread in the trajectory of a football. Think of a straight path, a mass of half a kilogram and a speed of 10 metres per second. Then the wavelength from equation (4) is approximately

$$\lambda = \frac{h}{mv} \approx \frac{7 \times 10^{-34}\ \mathrm{J\,s}}{0.5\ \mathrm{kg} \times 10\ \mathrm{m\,s^{-1}}} \approx 10^{-34}\ \mathrm{m}.$$

How wide a slit is necessary for a straight-line path (figure 2)? For a path length $2a = 20$ m, equation (1) gives us

$$d^2 \approx 10^{-33}\ \mathrm{m}^2$$

so the effective transverse spread of the path due to quantum effects is

$$2d \sim 10^{-16}\ \mathrm{m}.$$

In other words, the centre of the football follows essentially a single path, as Newton and everyday experience attest.

'*Explore all paths*' and the principle of least action

Equation (2) is important enough to be worth repeating here:

$$f = \frac{L}{h} = \frac{K - U}{h}.$$

It gives the rate of rotation of the quantum arrow along a path. Thus the sum (integral) of $(L/h)\,\mathrm{d}t$ along a path gives the total number of rotations along a path. The integral of $L\,\mathrm{d}t$ has a name and a long history. The Irish mathematician William Rowan Hamilton (1805–1865) formulated this integral, to which we give the name *action*. He showed that the classical path between two points fixed in space and time was always the path that had the least (or anyway, stationary) value for the action. He called this the *principle of varying action*. Most nowadays call it the *principle of least action*, not worrying about the fact that sometimes the action is stationary at a saddle point. The action along a path is just the sum of a lot of contributions of the form

$$L\,\mathrm{d}t = (K - U)\,\mathrm{d}t.$$

Feynman's crucial and deep discovery was that you can base quantum mechanics on the postulate that L divided by the quantum of action h is the rate of rotation of the quantum arrow. We have therefore started at that point, with the fundamental quantum command *Explore all paths!* We then went to the classical limit in which all paths contract toward one path and the quantum command is transformed into *Follow the path of least action!*

Least action explains Newton's laws of motion

We complete the transition to Newtonian mechanics by showing, by illustration rather than proof, that the principle of least action leads directly to Newton's second law: $F = \mathrm{d}p/\mathrm{d}t$. We adapt a simple way to do this described by Hanc *et al* (2003). We choose the special case of one-dimensional motion in a potential energy function that varies linearly with position. To be specific, think of a football rising and falling in the vertical y-direction near Earth's surface, so the potential

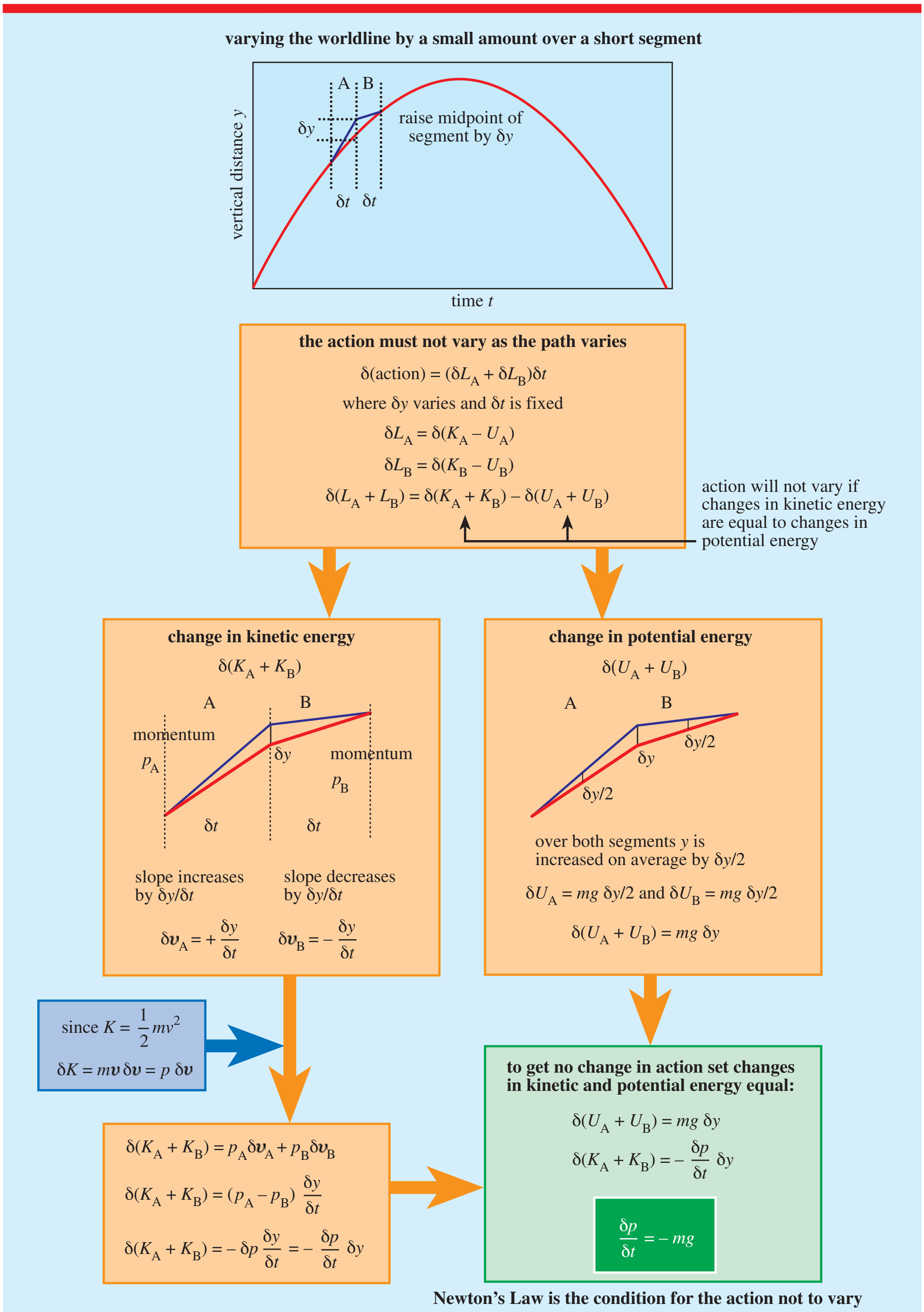

Figure 3. From least action to Newton.

energy function $U(y)$ is given by the expression mgy and the Lagrangian L becomes

$$L = K - U = \tfrac{1}{2}mv^2 - mgy$$

with the velocity v in the y-direction.

Now look at figure 3, which plots the vertical position of the centre of a rising and falling football as a function of time. This position–time curve is called the *worldline*. The worldline stretches from the initial position and time—the initial event of launch—to the final event of impact. Suppose that the worldline shown is the one actually followed by the football. This means that the value of the action along this worldline is a minimum.

Now use scientific martial arts to throw the problem onto the mat in one overhand flip: If the action is a minimum along the entire worldline with respect to adjacent worldlines, then it is a minimum along *every segment* of that worldline with respect to adjacent worldlines along that segment. 'Otherwise you could just fiddle with just that piece of the path and make the whole integral a little lower.' (Feynman 1964, p 19-8). So all we have to do is to ensure that the action is a minimum along any arbitrary small segment.

We take a short segment of the worldline and vary the y-position of its centre point, shifting it up by a small amount δy. Then we demand the condition that such a shift does not change the action along the segment (leaving the end-points fixed).

We consider the change in the action over the two parts A and B of the segment, which occupy equal times δt. The change in the action is

$$\delta S_{\mathrm{AB}} = \delta S_{\mathrm{A}} + \delta S_{\mathrm{B}} = (\delta L_{\mathrm{A}} + \delta L_{\mathrm{B}})\delta t.$$

Since δt is fixed, the only changes that matter are those in δL_{A} and δL_{B}. These are

$$\delta L_{\mathrm{A}} = \delta(K_{\mathrm{A}} - U_{\mathrm{A}}) \qquad \delta L_{\mathrm{B}} = \delta(K_{\mathrm{B}} - U_{\mathrm{B}}).$$

Rearrange the terms to write their sum as

$$\delta(L_{\mathrm{A}} + L_{\mathrm{B}}) = \delta(K_{\mathrm{A}} + K_{\mathrm{B}}) - \delta(U_{\mathrm{A}} + U_{\mathrm{B}}). \quad (5)$$

It remains to see how the two sums $K_{\mathrm{A}} + K_{\mathrm{B}}$ and $U_{\mathrm{A}} + U_{\mathrm{B}}$ change when the centre of the worldline is shifted by δy. We shall require the changes to be equal, so that the change in the action, which is their difference multiplied by the fixed time interval δt, then vanishes, and we know that the action is unvarying with respect to such a worldline shift.

Take first the change in potential energy. As can be seen in figure 3, the average change in y along both parts A and B due to the shift δy in the centre point is $\delta y/2$. The football is higher up by $\delta y/2$ in both parts of the segment of worldline. Thus the total change in potential energy is

$$\delta(U_{\mathrm{A}} + U_{\mathrm{B}}) = mg\,\delta y. \quad (6)$$

Thus we have the change in the sum of potential energies, i.e. the second term in equation (5). An easy first wrestling move!

Getting the first term, the change in the sum of kinetic energies, needs a trifle more agility. The effect of the shift δy in the centre point of the worldline is to increase the steepness of the worldline in part A, and to *decrease* it in part B (remember the end points are fixed). But the steepness of the worldline (also the graph of y against t) is just the velocity v in the y-direction. And these changes in v are equal and opposite, as can be seen from figure 3. In parts A and B the velocity changes by

$$\delta v_{\mathrm{A}} = \frac{\delta y}{\delta t} \qquad \delta v_{\mathrm{B}} = -\frac{\delta y}{\delta t}.$$

But what we want to know is the change in the kinetic energy. Since

$$K = \tfrac{1}{2}mv^2$$

then for small changes

$$\delta K = mv\,\delta v = p\,\delta v.$$

Thus we get for the sum of changes in kinetic energy

$$\delta(K_{\mathrm{A}} + K_{\mathrm{B}}) = p_{\mathrm{A}}\delta v_{\mathrm{A}} + p_{\mathrm{B}}\delta v_{\mathrm{B}}$$

or, remembering that the changes in velocity are equal and opposite,

$$\delta(K_{\mathrm{A}} + K_{\mathrm{B}}) = (p_{\mathrm{A}} - p_{\mathrm{B}})\frac{\delta y}{\delta t}.$$

The change in momentum from part A to part B of the segment is the change $\delta p = p_{\mathrm{B}} - p_{\mathrm{A}}$. Thus the total change in kinetic energy needed for equation (5) is simply

$$\delta(K_{\mathrm{A}} + K_{\mathrm{B}}) = -\frac{\delta p}{\delta t}\,\delta y. \quad (7)$$

Now for the final throw to the mat! Equation (5) says that if the changes of kinetic energy (from equation (7)) are equal to the changes in the potential energy (from equation (6)) then their difference is zero, and the action does not change (for fixed δt). Thus we must equate the two to find the condition for no change in the action:

$$-\frac{\delta p}{\delta t}\,\delta y = mg\,\delta y.$$

That is, the change in the action is zero if

$$\frac{\delta p}{\delta t} = -mg.$$

In the special case chosen, mg is the gravitational force in the negative (downward) direction. In the limit of small segments this result becomes

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} \qquad \textbf{Newton's Second Law}.$$

It's all over: the problem lies at our feet! The force *must be* equal to the rate of change of momentum. Newton's law is a *consequence* of the principle of least action, which is itself a consequence of quantum physics.

What about a more general potential energy function? To begin with, *every* actual potential energy function is effectively linear for a small increment of displacement. So the above analysis still works for a small enough increment along *every* small segment of *every* actual potential energy function. By slightly modifying the derivation above, you can show that the general case leads to

$$-\frac{\mathrm{d}U}{\mathrm{d}y} = \frac{\mathrm{d}p_y}{\mathrm{d}t}$$

where $-\mathrm{d}U/\mathrm{d}y$ is the more general expression for force, and we have added the subscript y to the momentum, since the motion took place up and down along the y-axis. For three-dimensional motion there are two more equations of similar form, one for the x-direction and one for the z-direction (and, to be technically correct, the derivative of U becomes a partial derivative with respect to that coordinate).

"I have been saying that we get Newton's law. That is not quite true, because Newton's law includes nonconservative forces like friction. Newton said that ma is equal to any F. But the principle of least action only works for conservative systems—where all forces can be gotten from a potential function." (Feynman 1964, p 19-7). Friction dissipates organized mechanical energy into disorganized internal energy; we are not trying to explain thermodynamics in this article!

Classical and quantum?

Let's get away from the algebra and try to describe how it all works at the fundamental level. Newton's law fixes the path so that changes in phase from changes in kinetic energy exactly match those from changes in potential energy. This is the modern quantum field theory view of forces: that forces change phases of quantum amplitudes. We see it here in elemental form. What Newtonian physics treats as cause and effect (force producing acceleration) the quantum 'many paths' view treats as a balance of changes in phase produced by changes in kinetic and potential energy.

So finally we have come all the way from the deepest principle of nonrelativistic quantum mechanics—*Explore all paths!*—to the deepest principle of classical mechanics in a conservative potential—*Follow the path of least action!* And from there to the classical mechanics taught in every high school. The old truths of the classical world come straight out of the new truths of the quantum world. Better still, we can now estimate the limits of accuracy of the old classical truths.

Half-truths we have told

In this article we have deliberately stressed an important half-truth, that every quantum object (a photon, an electron etc) is significantly like every other quantum object: namely, that all obey the same elemental quantum command *Try everything!* But if electrons as a group behaved exactly like photons as a group, no atom would exist and neither would our current universe, our galaxy, our Earth, nor we who write and read this article.

The *Try everything!* half-truth does a good job of describing the motion of a single photon or a single electron. In that sense it is fundamental. But the behaviour of lasers and the structure of atoms depend respectively on collaboration among photons and collaboration among electrons. And collaboration is very different between electrons and between photons. Photons belong to the group *bosons*, electrons to the group *fermions*. Bosons tend to cluster in the same state; fermions avoid occupying the same state.

If two identical particles come to the same final state, the same result must come from interchanging the two particles—that is the symmetry consequence of identity. The *Try everything!* command has to include the command 'Add up the arrows for both processes.' If the particles are bosons, the arrows are the same and just add, doubling the amplitude (and multiplying the probability by four). We summarize by saying that photons 'like to be in the same state'; this is why lasers work (and also why we experience radio-frequency streams of photons as if they were radio waves). But if the particles are fermions, the arrows combine with *reversal* of phase. Now the amplitude to be in the same state is *zero*. That's why electrons obey the Pauli exclusion principle; why electrons in an atom are in different states. The structure of our world and our observation of it both depend on this difference between the group behaviour of photons and the group behaviour of electrons.

Acknowledgments

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Appendix. The de Broglie wavelength

We show here that the fundamental expression L/h for the rate of rotation of the quantum arrow as a particle propagates along a path, leads to the de Broglie relationship $\lambda = h/mv$, in a suitable approximation.

We consider a free particle, where the potential energy $U = 0$ and $L = K = \frac{1}{2}mv^2$:

$$\text{Rate of rotation of arrow} = \frac{mv^2}{2h}.$$

Over a time t, the number of rotations of the arrow is

$$\text{Number of rotations } n = \frac{mv^2}{2h}t.$$

One wavelength corresponds to the *distance* x along which the arrow makes one complete turn. So we need to express the number of rotations in terms of x rather than v and t, using $v = x/t$. This gives

$$\text{Number of rotations } n = \frac{mx^2}{2ht}.$$

If x is large and increases by a small amount δx, the number of rotations increases by

$$\delta n = \frac{mx\,\delta x}{ht} \quad (\text{neglecting terms in } \delta x^2).$$

We now introduce the wavelength. Provided that $\lambda \ll x$, we can say that $\delta n = 1$ rotation when $\delta x = \lambda$. That is

$$1 = \frac{mx\lambda}{ht}.$$

Writing $v = x/t$ and rearranging gives the de Broglie relationship

$$\lambda = \frac{h}{mv}.$$

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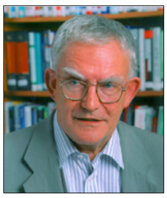

Jon Ogborn directed the Institute of Physics' *Advancing Physics* project, and is Emeritus Professor of Science Education, Institute of Education, University of London.

Edwin Taylor is a retired member of the Physics Department at the Massachusetts Institute of Technology and coauthor of introductory texts in quantum mechanics, special relativity and general relativity as well as interactive software to learn these subjects.